Taylor Columns Tank Experiment

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Introduction

Sir George Ingram Taylor, a physicist and mathematician who made notable contributions in fluid dynamics and wave theory, first predicted Taylor columns in 1923. His predictions were based on the theorem proved in 1915 by the oceanographer and mathematician, Joseph Proudman. The combined research of these two scientists is embodied in what is now called the Taylor-Proudman Theorem. In the paper published in 1923, Taylor observed that, a slowly moving object (in rotation) behaves nearly as a solid cylinder extended parallel to the rotation axis and that this could be applied to motions in atmospheres and oceans. The experimental setup included creating a simplified experiment to show the concepts Taylor observed in his own two concentric cylinder tank experiment (Image 1, Taylor 1923).

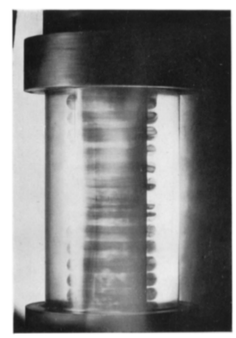

Image 1

Theory

The Taylor-Proudman Theorem is defined as when a solid body moves slowly through a steadily rotating fluid (at high angular velocity), the fluid velocity will be uniform throughout the fluid for any line parallel to rotational axis. For the theorem to hold, certain conditions must be met.

First, the Taylor-Proudman theorem states that the rotating fluid must react solid-body rotation. Solid-body rotation can be described as a fluid that rotates at a constant angular velocity. The fluid increases in rigidity and acts more solid-like. To maintain solid-body rotation, momentum must be conserved. The Navier-stokes equation represents the conservation of momentum of a fluid in motion. For a rotating fluid, the Navier-Stokes equation is as such:

$$(2\Omega \times u) + \left(\frac{1}{\rho}\right)\nabla P + \nabla\varphi = \upsilon\nabla^2 u + \frac{\delta u}{\delta t} + (u \cdot \nabla)u$$

Second, the rotating body should have acceleration mainly due to the Coriolis force, and not due to inertia. By keeping the angular velocity large compared to the movement of the fluid (slow and steady flow), acceleration due to Coriolis dominates the system. If the flow was fast, inertia acceleration would have a greater impact on the movement of the fluid and a centrifugal force would accelerate the water away from the rotating center. By maintaining a slow flow, the fluid can only deflect in the direction of the angular velocity (Image 2). By taking all assumption into account, the equation for the conservation of momentum of a rotation fluid is reduced to:

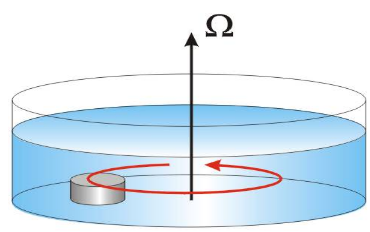

Image 2

$$2\Omega \times u = 0$$

where the acceleration due to the Coriolis force is left equal to zero. By solving the equation:

$$2\Omega \cdot \nabla u = 2\Omega\frac{\delta u}{\delta z} = \frac{\delta u}{\delta z} = 0$$

it can be stated that the change in velocity with respect to height is zero. This supports the definition of Taylor column stating that the fluid velocity will be uniform throughout the fluid for any line parallel to rotational axis.

For a Taylor column to form within a rotating fluid, a solid object must be submerged in the rotating liquid. In order for a Taylor column to form, the submerged obstacle at the bottom of the rotating tank (puck in Image 3) requires a height much less than the height of the rotating fluid. The fluid at the bottom (same height as the puck) flows

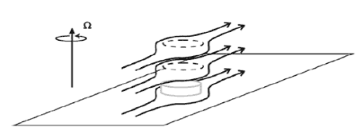

Image 3

around the object as the fluid rotates around it (Image 3). Since the fluid is in solid-body rotation, the flow around the puck extends in the direction of the rotation vector (vertical).

Experimental Setup, Troubleshooting and Results

Preliminary experiments were conducted in Ritter 229, access to the room after hours and weekends was achieved by borrowing a key from the TA (Madeleine Hamann). A white plastic square plate was placed on top of the turntable to serve as a white background to help contrast the dyes used in the experiment. A long flexible clear plastic was clipped into the shape of a cylinder whose diameter matched the length of a side of the square tank for creating a cylindrical body of water. The tank was positioned over the white square plate, ensuring that the corners sat evenly above the four marked positions of the turntable. The tank was filled to 3 inches below the edge of the cylinder with tap water by connecting a rubber hose to faucet head. Then the rotating turntable-cart was position with space on all sides to reduce bumping and vibrations from adjacent objects, and power was connected for the rotating table motor and camera. Additionally, a remote controlled GoPro Hero 3 was clamped on the post alongside the turntable camera in order to record footage of the experiment. A hockey puck was used as the obstacle and was placed about 2 inches from the edge of the cylinder. The tank was allowed to reach solid body rotation for at least 20 minutes for every attempt made to generate a Taylor column.

It took several attempts to generate a Taylor column, and various combinations of solid body rotation velocity and changes in velocity were tried. To remove as many variables that could contribute to inertial forces, the windows were closed and the vibrating power supply was removed from the tank experiment table. The experiment was attempted at the fastest speed, but that of course introduced too much inertial forces and the dye could be seen feeling the effects of centrifugal force.

In end, the optimized condition used for successfully generating a Taylor column in this experiment was starting with a rotational speed of 44 on the bike odometer, which was calculated to be about 4.4 rpm. After 20 minutes of rotation, the water was assumed to be in solid body rotation. Then, dye droplets were added behind the puck. The turntable was then carefully slowed to a speed of 42 or 4.2 rpm; this required some skill, as the knob is quite sensitive. This action quickly slows down the speed of the tank and the puck, while inertia of water wants to maintain the same speed. This moves water across the X-Y plane of the puck and a Taylor column is generated and visualized by the dye.

Real Life Example

As a potential example of a real world Taylor column, the Chukchi Sea located north of the Bering Strait, and between Alaska and Siberia, is home to the Herald and Hanna Shoals (Image 4). During the summer, warmer waters flow in through the Bering Strait and encounter the sea ice, which begins to melt. However, there is an area over the sea in which ice does not melt away, and this ice sits above the Herald and Hanna Shoals. It is hypothesized that this could be due Taylor columns forming above the two shoals, which insulates the ice at the surface from the warmer waters (Martin, Seeke, and Drucker 1997).

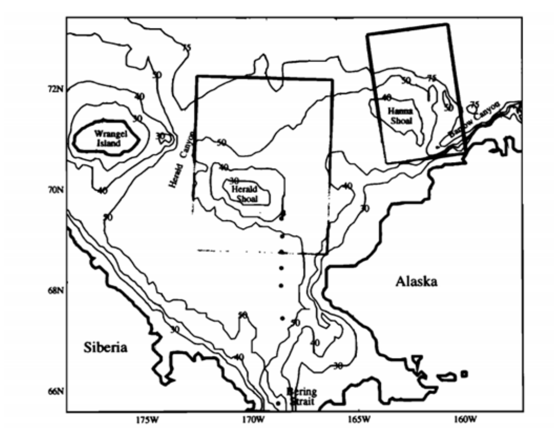

Image 4

Conclusion

Sir George Ingram Taylor predicted the existence of Taylor columns in nature almost a century ago. The oceanographic example explained above is one of many Taylor column examples that is a 'potential' Taylor column. Taylor columns are relatively easy to demonstrate in a controlled setting, such as the rotating tank experiment performed for this report, because forces (friction/viscosity, pressure gradient force, advection) that prevent solid body rotation from being attained are diminished. On Earth, there are a countless number of forces constantly acting on the oceans. Therefore, a Taylor column may be observed in nature, however it may be even more difficult to prove the observation was a 'real' Taylor column. Therefore, tank experiments are very valuable such that they can help visualize an ocean phenomenon that may rarely occur in nature.

Symbol List

Ω……………..Rotational Vector
ρ……………...Density
P……………...Pressure
t………………Time
u……………...Viscosity
u……………...Angular/Rotational Velocity
φ……………...Gravitational Potential
∇……………..Nabla Operator

References

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Michael Brenner *Rotating Fluid Dynamics: Taylor-Proudman Theorem* *(*www.math.mit.edu/brenner/18.354/handouts/lect13/node2.html)

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Contributions

Mark: Theory of Taylor Column and the math behind it in presentation and report, conclusion.

Chang: Introduction in presentation and report, explanation of methods in presentation, troubleshooting component in report.

Daniel: GoPro footage recording and editing, real life Taylor-Columns example in the presentation and report, fine knob-speed adjustment, wikipage upload.