SIO 210 Problem Set 2
October 17, 2019
Due October 24, 2019 (1 week)

If you work together on these, please make sure that you understand the concepts and use your group discussion to help with your understanding.

1. (a) The California Current is a oceanic region along the west coast of North America. The upwelling region of the California Current is closest to the coast. This region is approximately 2000 km long along the coast and 100 km wide off the coast. Suppose that there is a net heating of this entire strip at a rate of 75 W/m^2 in the annual mean. If this heat is applied to the top 100 m of the ocean, calculate how much it will warm in one year. Assume that density is approximately 1025 kg/m3 and specific heat is approximately 4000 J/(kg °C). Assuming that the initial temperature is 10°C, what is the final temperature?). Assume that this layer is well mixed. Assume there is no volume transport into or out of the layer.

dT/dt = [1/(rho c_p)]Q/h
dT = dt*Q/(rho c_p h)
= (1 year)*(60sec/min * 60 min/hr * 24 hr/day * 365 day/yr) *(75 W/m^2) /[1025 kg/m3 * 4000 J/(kg°C) *100 m] = 5.77°C

Which information given above is irrelevant to your answer to this part?
The length and width information (2000 km and 100 km) are irrelevant. The calculation applies to unit area.

(b) Now consider this as a steady state balance. In the California Current, water flows in towards the shore, upwells along the coast, warms, and flows offshore at the sea surface. Assume that the inflow is at 8°C, and the surface outflow is at 12°C.

Using information given in (a), calculate the overturning (upwelling) volume transport in this system.
Note: Unlike in (a), there IS a volume transport into and out of the CA Current. This calculation considers only the layers moving into the upwelling region and moving out of the upwelling region. You do not look at the actual process within the heating region, only at the total amount of heat applied, with an assumption that processes within the heating strip changes the inflow into the outflow.
It might be helpful to make a sketch of this.
$V = V_i = V_o$. Solve for V.

Let delT = (T_o – Ti)
V*delT*(rho c_p) = A_sQ
As = (2000km)*(1000m/km)*(100km) *(1000 m/km) = 2 x 10^{11} m^2

V = $(A_sQ)/(\rho\ c_p\ \text{del}T) = [2 \times 10^{11}\ m^2 * (75W/m^2)]/[1025\ kg/m3 * 4000\ J/(kg°C) * 4°C] = 9.14 \times 10^5\ m^3/sec = 0.91 \times 10^6\ m^3/sec = 0.91\ Sv$

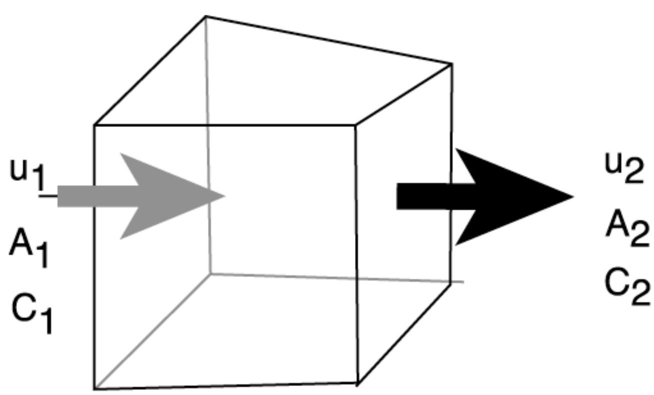
Which information given in (a) is irrelevant for this part?
The elapsed time of 1 year.

(c) Farther north along the coast, off Canada and Alaska in the Alaska Current, there is net precipitation.
Note: net precipitation means that F = –(R + AP) + AE is net negative.

If there is net precipitation of 100 cm/yr in a strip the same size and depth as in problem 3, calculate how much the water column will freshen in one year. Assume that the initial salinity is 33 psu. What is the final salinity?
The easiest way to solve this is to consider it as a dilution problem
At the start, h = 100m, S1 = 33 psu = 33 gm/kg seawater
At the end, h = 101 m, S2 = (100m/101m)(33 psu) = 32.67 psu

2. Consider the concentration in a volume of the ocean of a conservative tracer (call it chlorofluorocarbon or CFC). The units of concentration are (moles tracer)/(kg seawater), that is, moles/kg.

Consider flow of this tracer through the volume in the figure. There is flow into the left side of the volume and flow out of the right side of the volume. The inflow has a tracer concentration C_1 and velocity u_1. The outflow has a tracer concentration C_2 and velocity u_2. The area of the inflow is A_1 and the area of the outflow is A_2. (There is no flow across any other surface.)



u_1 = 0.1 m/sec	u_2 = ???
A_1 = 10 kilometers x 100 meters	A_2 = 5 kilometers x 100 m,
C_1 = 0.8 pmol/kg,	C_2 = 1.0 pmol/kg

ρ = 1025 kg/m^3

a) Calculate u_2

$A_1 = 10 \times 10^3 \times 10^2\ m^2 = 10^6\ m^2$
$V_1 = u_1 A_1 = (0.1\ m/sec) * 10^6\ m^2 = 10^5\ m^3/sec$

Since $V_2 = V_1$, $V_2 = u_2 A_2 = 10^5\ m^3/sec$
$A_2 = 0.5 \times 10^6\ m^2$
So $u_2 = 2 \times 10^5\ m^3/sec$

You can also notice that $u_2 = (u_1 A_1)/A_2 = u_1 (A_1/A_2)$ and $A_1/A_2 = 2$

b) What is the FLUX of tracer on the left side (where the values are C_1, u_1, A_1)?
Flux = $\rho u_1 C_1 = (1025\ kg/m^3)(0.1\ m/sec)(0.8\ pmol/kg) = 82.0\ pmol/(m^2\ sec)$

c) What is the TRANSPORT of tracer on the left side (C_1, u_1, A_1)?
Note that $V = V_1 = V_2$
$\rho V C_1 = (1025\ kg/m^3)(10^5\ m^3/sec)(0.8\ pmol/kg) = 0.82 \times 10^8\ pmol/sec$

d) What is the TRANSPORT of tracer on the right side (C_2, u_2, A_2)?
$\rho V C_2 = (1025\ kg/m^3)(10^5\ m^3/sec)(1\ pmol/kg) = 1.025 \times 10^8\ pmol/sec$

e) compare answers (c) and (d). What do you think is happening to the tracer within the volume? (CFCs for instance are conservative tracers, and come from the atmosphere.)
There is a source of tracer into the volume since the exit concentration is higher than the entrance concentration.

3. The Pacific Ocean is approximately 10,000 km wide. Its upper layer ("wind-driven gyre") is approximately 1,000 m deep. Consider a west-to-east cross-section at about 24°N across the whole width of the Pacific, from Asia to North America, for this layer. Assume that there is a narrow northward western boundary current (Kuroshio) and a very broad southward flow across most of the section. For the following questions, assume that velocity does not vary with depth within this layer.

(a) If the southward flow is 1 cm/sec, calculate the total southward volume transport, in MKS units. (Ignore the western boundary current for this calculation.)
Area (vertical cross-section): $A = 10000\ km \times 1000\ m = (10^4 km) \times (10^3\ m/km) \times 10^3 m = 10^{10}\ m^2$ (ignoring that there is a boundary current of about 100 km, which is a very small fraction of the entire ocean width).
Volume transport $V = v \times A = (1\ cm/sec) \times (10^{-2}\ m/cm) \times 10^{10}\ m^2 = 10^8\ m^3/sec = 100 \times 10^6\ m^3/sec = 100\ Sv$.

(b) If this same amount of water returns northward in a western boundary current that is 100 km wide (and still 1 km deep), calculate the average northward velocity of the western boundary current.

Northward volume transport must equal southward volume transport (ignoring Bering Strait). Can just do this proportionally: $V_{north} = V_{south} (A_{south}/A_{north}) = V_{south} \times (10{,}000/100) = V_{south} \times 10^2 =$ 100 cm/sec.

(c) If the average oxygen concentration of the northward flow in the western boundary current is 150 μmol/kg, calculate the net northward transport of oxygen in the western boundary current, in units of μmol/sec. Use the information from (b) to calculate. Assume density is rho = 1025 kg/m^3.

T_{oxygen} = rho * oxygen content * volume transport = 1025 kg/m^3 * 150 μmol/kg * 100 x 10^6 m^3/sec = 1.57 x 10^{13} μmol/sec = 1.57 x 10^7 mol/sec

(d) Suppose this circulation transports 1 PW of heat northward. If all of the northward flow is of one temperature and all of the southward flow is of another temperature, what is the temperature difference between the northward and southward flow? Use typical (uniform) values for density and specific heat, as given in class or in a textbook.

Heat transport = 1 PW = 1 x 10^{15} W = V(rho c_p) ($T_{north} - T_{south}$) = (10^8 m^3/sec) (1025 kg/m^3)(4000 J/kg°C) ($T_{north} - T_{south}$)

Therefore ($T_{north} - T_{south}$) = (1 x 10^{15} W)/[(10^8 m^3/sec) (1025 kg/m^3)(4000 J/kg°C)] = 2.44°C

(e) Explain why I asked you to calculate a temperature difference in (d), rather than the actual temperature.

The northward heat transport is balanced by surface heat loss over the whole Pacific north of that section. This changes the temperature, but gives no information about its absolute value. Also, I would not have asked you to calculate the heat transport by a particular current, such as the Kuroshio, because heat is defined relative to the Kelvin scale, where absolute 0 K is at -273.16°C.

(f) Calculate the average air-sea heat flux between this section and the northern edge of the Pacific Ocean (Japan/Russia/U.S./Canada). Use very simplified assumptions about the width and length of this region (i.e. don't worry about calculating the exact dimensions, just approximate it). (Ignore Bering Strait – assume there is no leakage out of this large "box".)

Need surface area of ocean for this: meridional x zonal distances

Assume shape is something like a rhombus.

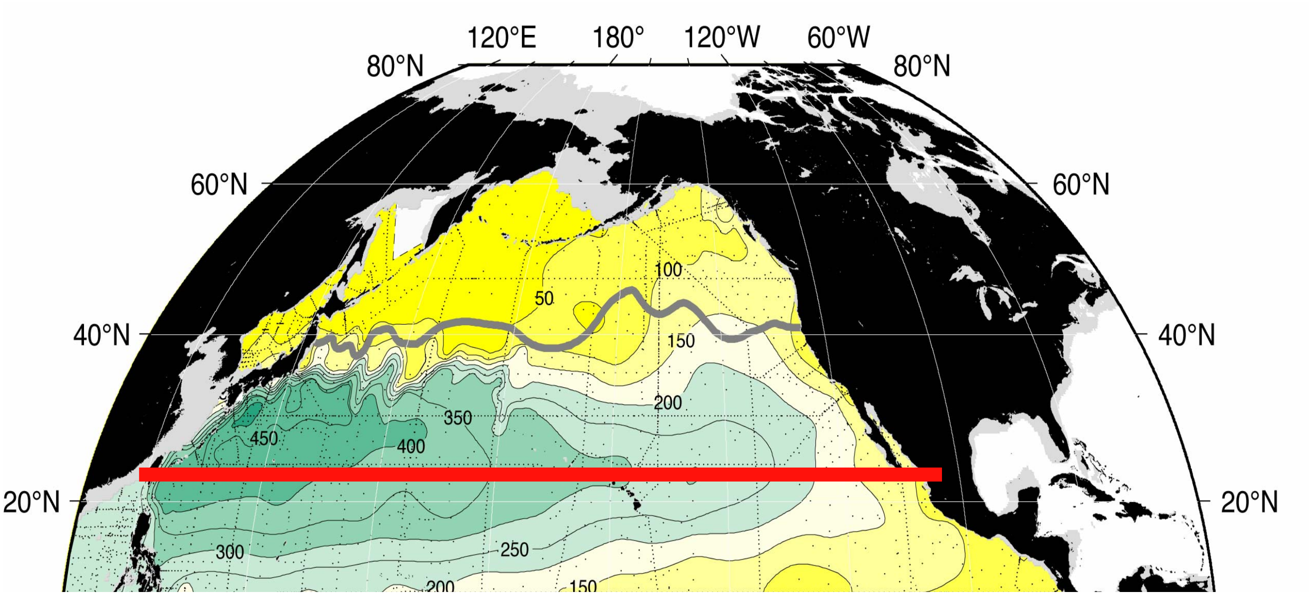
Meridional: About 40°latitude = 40 x 111 km/deg latitude =4440 km = 4.44 x 10^6 m

Zonal: About 70°longitude at 20°N. From the WOCE atlas (http://whp-atlas.ucsd.edu/pacific/p03/sections/printatlas/printatlas.htm), we can see that this section is 12,500 km long, or you can estimate it from the latitude and sin(latitude). Farther north, e.g. 50°N, width is more like 7000 km wide. Approximate area is therefore 9500 km x 4440 km = 9.5 x 4.4 x 10^6 x 10^6 m^2 = 40.5 x 10^{12} m^2

Heat that is lost is 1 PW = 1 x 10^{15} W.

Therefore average heat flux (loss) is 1 PW/Area = 1×10^{15} W/(4.05×10^{13} m^2) = 0.24×10^2 W/m^2 = 24 W/m^2

Value that you get will depend on how you've approximated the surface area, so there is no perfect answer, but it should be this order of magnitude.



4. Suppose you measure temperature at the end of the SIO pier at the same time every day for ten years. Assume 1 year = 365 days (ignore leap years).

(a) Is this measurement Eulerian or Lagrangian? Define both terms.

This is an Eulerian measurement since it is done at a fixed location.

A Lagrangian measurement follows the flow.

(b) What is the Nyquist frequency for your time series?

Sampling interval is delta t = 1 day

Time series length is 10 years.

Nyquist frequency is the highest frequency that is resolved, so it is defined by the sampling interval.

It is $f_{Nyquist}$= 1/(2 delta t) = 1/(2 days).

That is, the shortest period that can be resolved is 2 days.

(c) What is the fundamental frequency for your time series?

$f_{fundamental}$ = 1/T = 1/10 years

This is the lowest frequency that can be resolved. Longest period is 10 years.

(d) Will you be able to resolve the seasonal cycle for this time series? Explain.

Yes, it will be well resolved, since a season is about 90 days, which is much longer than the Nyquist period, and there will be 10 years of seasonal cycles, so the uncertainty in estimating the power at the seasonal period will be low.

(e) Will you be able to resolve the diurnal cycle (daily cycle) with this time series? Explain.
No, we can't resolve the diurnal cycle. We would need to sample at least twice per day for the Nyquist period to 1 day, and we would need to resolve at least several cycles per day for the information to be significant.

(f) Will you be able to detect significant climate signals with this time series, assuming that climate time scales in the North Pacific are 15 years to 40 years?
No, we can't resolve climate time scales because the period for the fundamental frequency is 10 years, which means that longer time scales are not resolved.

(g) Will you be able to detect significant climate trends with this time series?
No, because we won't be able to distinguish a trend from decadal time scales, which are not resolved.

5. Diffusivity κ has units of (length)^2 / time.

a) The molecular diffusivity of temperature in water is 0.0014 cm^2 /sec. Approximately how long would it take for temperature to diffuse 50 meters? You do not need an elaborate equation or to solve anything. Just use a dimensional argument based on units of κ.
We use a simple dimensional argument:
Units of κ are L^2/T, where L is length and T is time.
Convert κ to units of m^2/sec: $\kappa = (1.4 \times 10^{-3}\ \text{cm}^2/\text{sec})*(1\text{m}/100\ \text{cm})^2 = 1.4 \times 10^{-7}\ \text{m}^2/\text{sec}$
Therefore

$$T \sim L^2/\kappa = (50\text{m})^2/(1.4 \times 10^{-7}\ \text{m}^2/\text{sec}) = (25\text{x}10^2/1.4\text{x}10^{-7})\ \text{sec} = 1.78 \times 10^{10}\ \text{sec}$$

Convert to years to get better idea of how long this is (not necessary for the homework):
1 year = 365*24*60*60 sec = 3.15 x10^7 sec
Therefore

$$T = 1.78 \times 10^{10}\ \text{sec}/(3.15 \text{x}10^7\ \text{sec/year}) = 566\ \text{years}$$

b) Describe very briefly (1-2 sentences) what we mean by "eddy diffusivity".
Eddy diffusivity is the effective diffusivity of a fluid or gas that arises from turbulent motions that move properties around, generally at much faster rates than molecular diffusivity. This makes mixing much more efficient.

c) What is a common source of vertical eddy diffusivity? (what can cause it?)
Vertical eddy diffusivity arises from fluid motions with vertical components, such as internal waves. Turbulence in an internal wave results from its breaking. Therefore breaking internal waves are a particularly effective mechanism for vertical eddy diffusivity.

d) What is a common source of horizontal eddy diffusivity? (what can cause it?)
Horizontal diffusivity results from horizontal motions that are turbulent. Unstable currents generate mesoscale eddies that act to stir fluid horizontally. Not discussed in class, but submesoscale motions (< 1 km horizontally) could be even more efficient.

e) Approximately how large are vertical and horizontal eddy diffusivities?
They have a range (see lecture notes): horizontal eddy diffusivity is 1 to 10^4 m^2/sec
Vertical eddy diffusivity is 10^{-5} to 10^{-4} m^2/sec.

f) Approximately how long would it take for temperature to diffuse 50 meters if it diffuses through vertical eddy diffusivity?

$T \sim L^2/\kappa = (50m)^2/(10^{-4}\ m^2/sec) = 2.5 \times 10^7$ sec = 0.79 years = 9.5 months

g) Approximately how long would it take for temperature to diffuse 50 kilometers if it diffuses through horizontal eddy diffusivity?
I gave a large range of possible horizontal eddy diffusivities, because the processes that create this turbulence are so varied. Look at time scale for the two extreme values:

$T \sim L^2/\kappa = (50\ km)^2 \times (10^6\ m^2/km^2)/(1 m^2/sec) = 2.5 \times 10^9$ sec = 79 years

To

$T \sim L^2/\kappa = (50\ km)^2 \times (10^6\ m^2/km^2)/(10^4 m^2/sec) = 2.5 \times 10^5$ sec = 0.0079 years = 2.9 days