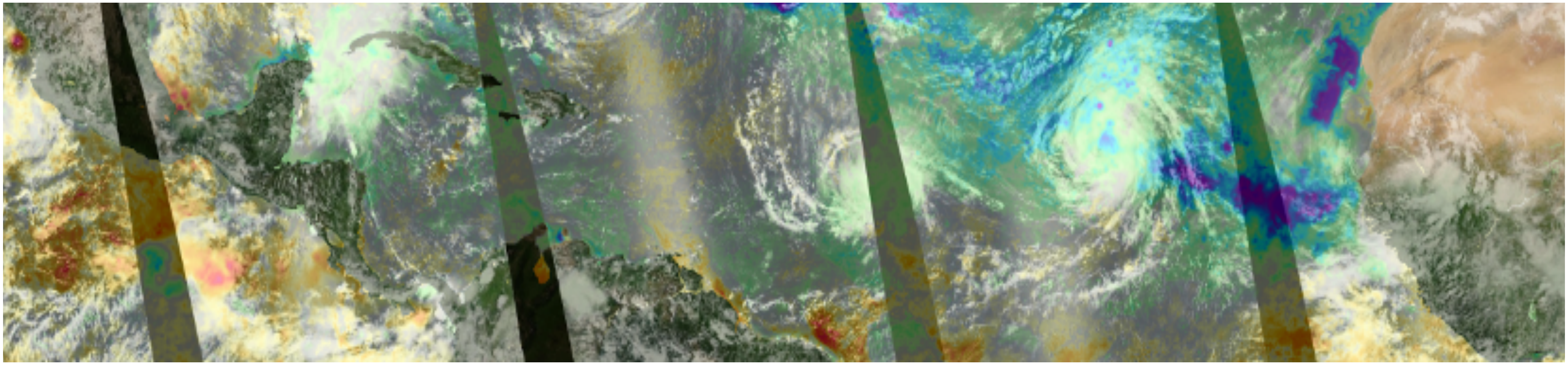


SIO 210: Dynamics I: transports, continuity, salt, heat

Radiation, Advection, Diffusion

Flux, transport

Conservation of volume

Continuity (a governing equation)

Conservation of salt

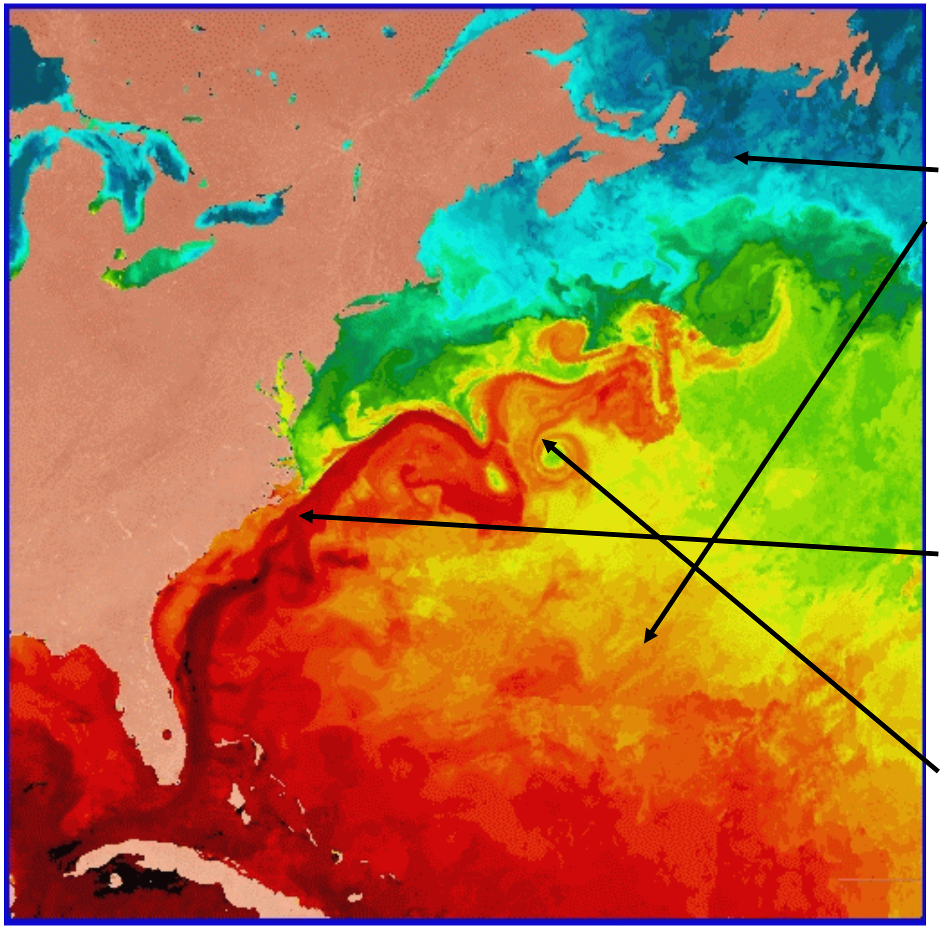
Freshwater transport

Heat budget

Heat transport

Reading: DPO Chapter 5.1, 2, 3, 4

Transport processes: radiation, advection, diffusion



(1) The surface heat balance, including radiation, makes the ocean warmer to the south, colder to the north (Northern Hemisphere).

(2) The Gulf Stream flows northward, advecting warm water.

(3) Eddies diffuse the heat.

Radiation, Advection, Diffusion

- Radiation: electromagnetic waves carry heat energy - sunlight, infrared radiation
- Advection: carry properties in currents
- Diffusion: moves properties through random motions, so somewhat similar to advection

Convergence and divergence of property fluxes can change local property

- Advective change: convergence or divergence of the property flux
- Diffusive change: convergence or divergence of the diffusive flux

[Next lecture: advection and diffusion in detail]

Flux: definition

- Flux of a property is associated with a **point in space**

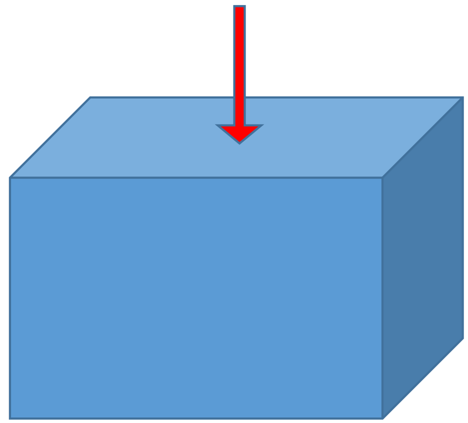
Velocity x density x concentration

Flux = $v\rho C$

Units of

(m/sec)(kg/m^3)(moles/kg) = moles/(sec m^2)

(Flux is the same as Transport per unit area – next slide)



Special case:

Mass flux is
velocity x density
Units are
(m/sec)(kg/m^3) = kg/(sec m^2)

Transport: definition

Transport = Velocity times density times concentration, integrated (summed) over area normal to the velocity

Transport = ∬vρCdA

units of

(m/sec)(kg/m^3)(moles/kg) m^2 = moles/sec

(same as Flux normal to an area integrated over that area.)

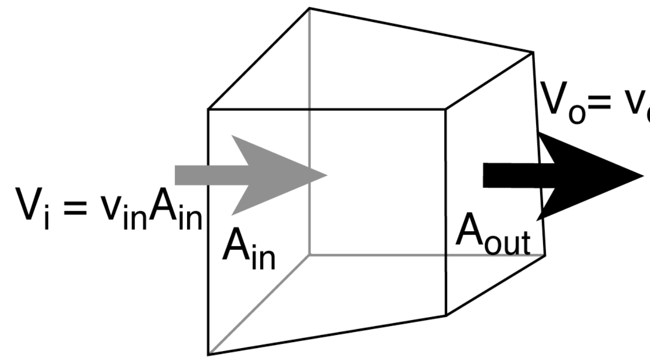


Figure 5.2

Special cases:

Volume transport is velocity times area
Units are
(m/sec) m^2 = m^3/sec

Mass transport is velocity x density x area
Units are
(m/sec)(kg/m^3)m^2 = kg/sec

Transport: more definitions

- Transport: add up (integrate) velocity time property over the area they flow through (or any area - look at velocity "normal" to that area)

- Volume transport = integral of velocity v — m^3/sec
- Mass transport = integral of density x velocity ρv — kg/sec

- Heat transport = integral of heat x velocity $\rho c_p T v$ — J/sec=W
- Salt transport = integral of salt x velocity $\rho S v$ — kg/sec
- Freshwater transport = integral of Fwater x velocity $\rho(1\text{-}S)v$ — kg/sec
- Chemical tracers = integral of tracer concentration (μmol/kg) x velocity $\rho C v$ — moles/sec

- (Flux is these quantities per unit area)

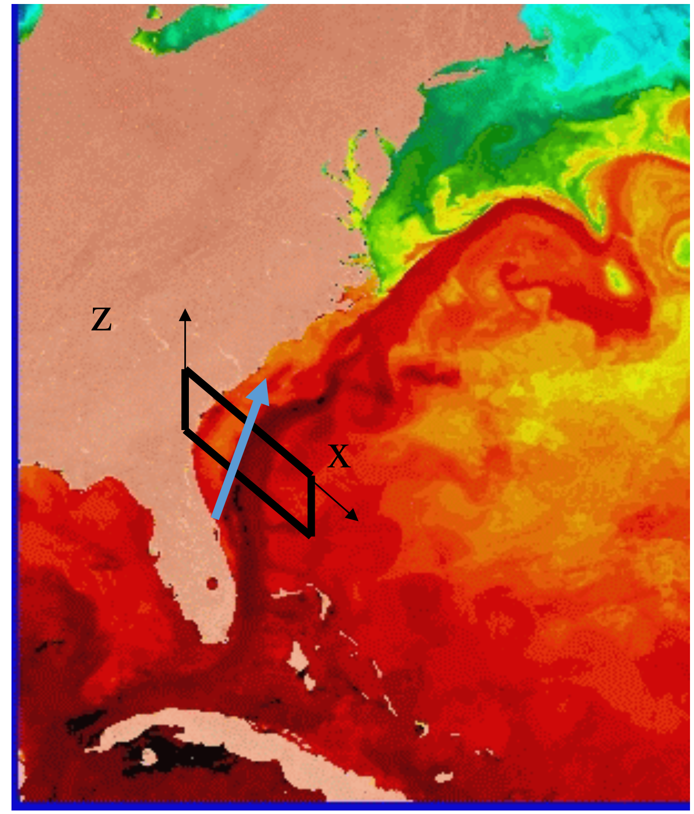
Transport definitions: quantitative

- Volume transport = $V = \sum v_i A_i = \iint v dA$ — m^3/sec
- Mass transport = $M = \sum \rho\, v_i A_i = \iint \rho v dA$ — kg/sec
- Heat transport = $H = \sum \rho\, c_p T v_i A_i = \iint \rho c_p T v dA$ — J/sec=W
- Salt transport = $\mathscr{S} = \sum \rho\, S v_i A_i = \iint \rho S v dA$ — kg/sec
- Freshwater transport = $F = \sum \rho\, (1\text{-}S)\, v_i A_i = \iint \rho (1 - S) v dA$ — kg/sec
- Chemical tracers = $\mathscr{C} = \sum \rho\, C v_i A_i = \iint \rho C v dA$ — moles/sec

- Flux is these quantities per unit area
  e.g. volume flux is V/A, mass flux is M/A,
  heat flux is H/A, salt flux is $\mathscr{S}$/A, freshwater flux is F/A, chemical tracer flux is $\mathscr{C}$/A

Quantify transport resulting from advection #1

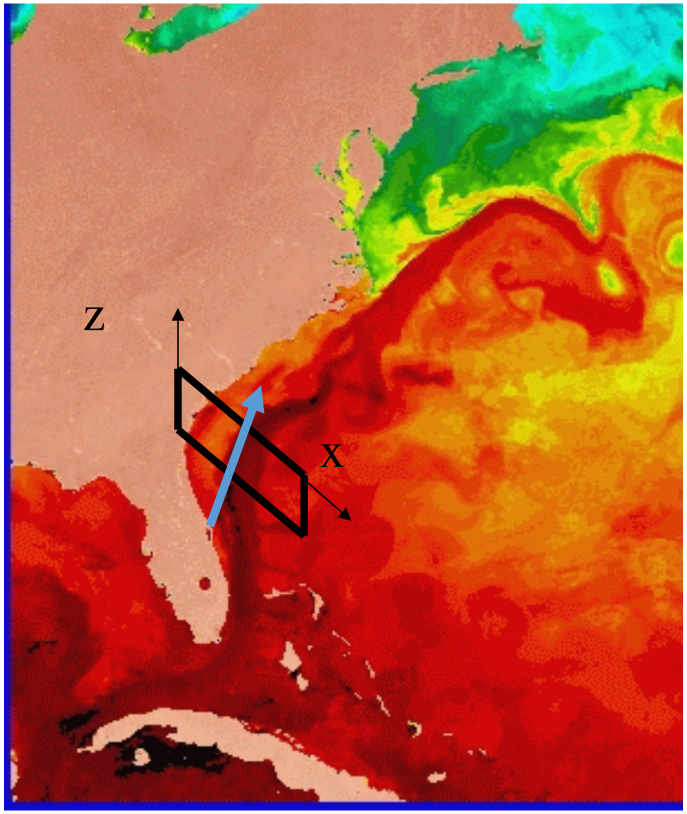
- Gulf Stream advects volume, mass, warm water northward
- How much water, how much mass is carried by the G.S past a certain point ?
- Draw a vertical plane across the current (x,z are across-stream and vertical)
- Measure current velocity at each point in the plane, normal to the plane
- Compute volume transport (velocity times area) for each small location in the plane and add them up (integrate) for total transport through the cross-section



Quantify transport resulting from advection #2

- Gulf Stream velocities are about 5 cm/sec at the bottom, up to more than 100 cm/sec at the top of the ocean. Assume an average of 20 cm/sec for this simplified (example) calculation.
- Assume a width of the current of 100 km
- Assume a depth of the current of 5 km
- The area across the G.S. is then 500 km^2
- Volume transport is then 20 cm/sec x 500 km^2

= 100 x 10^6 m^3/sec

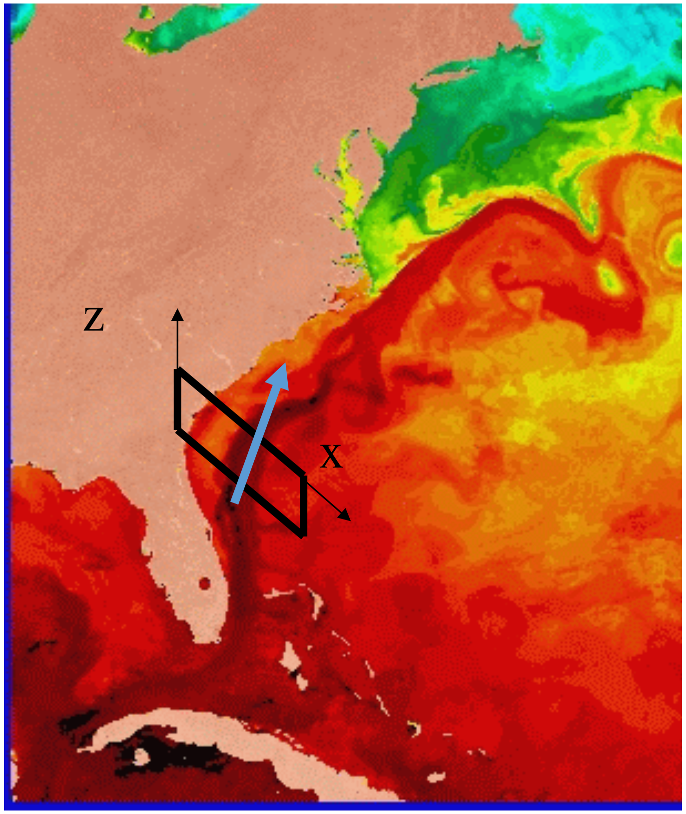


Define unit of volume transport:

Volume transport V: new unit

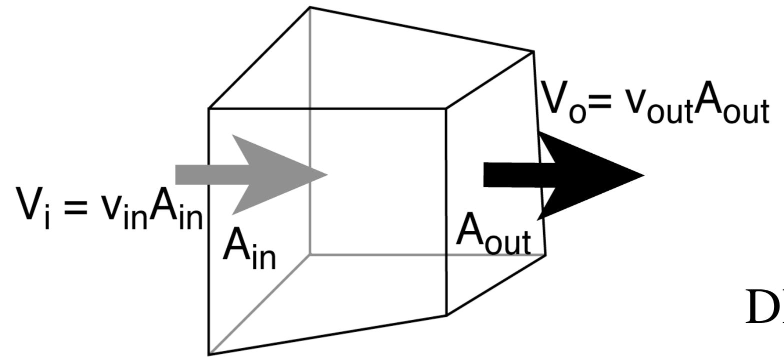
1 Sverdrup = 10^6 m^3/sec

So volume transport on previous slide is 100 Sv

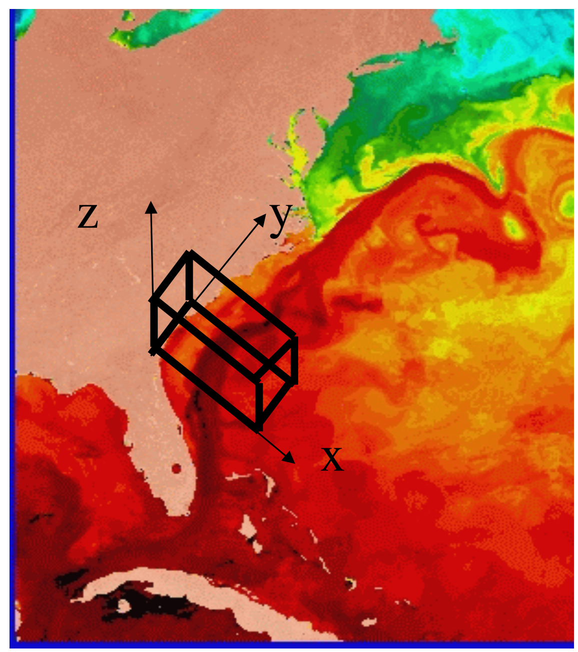


Conservation of volume: Continuity

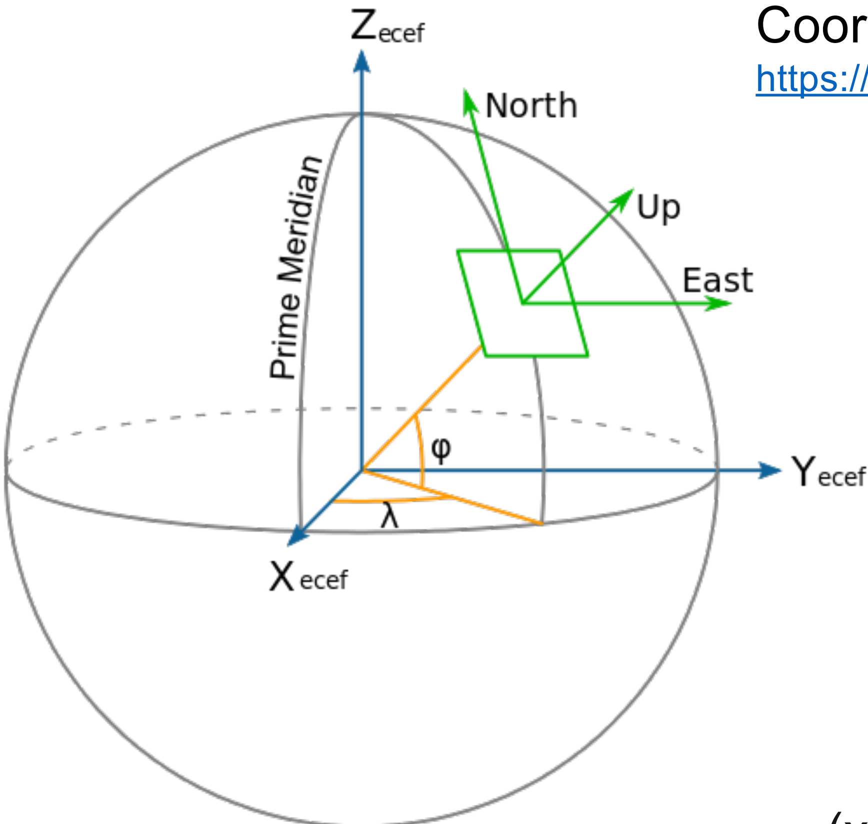
- Transport V into a box must equal the transport out of the box, adding up on all faces of the box.
- $V_i = V_o$
- (Including a very small residual for evaporation and precipitation, which is transport out top of box, if at sea surface.)
- Compute transport through each face of the volume. Total must add to 0
- (NO HOLES)



DPO Figure 5.2



Coordinate system: ‘Local’ Cartesian coordinates



Coordinate system: “East-North-Up”
https://en.wikipedia.org/wiki/Geographic_coordinate_system

Green: local Cartesian coordinate frame

x is East
y is North
z is up

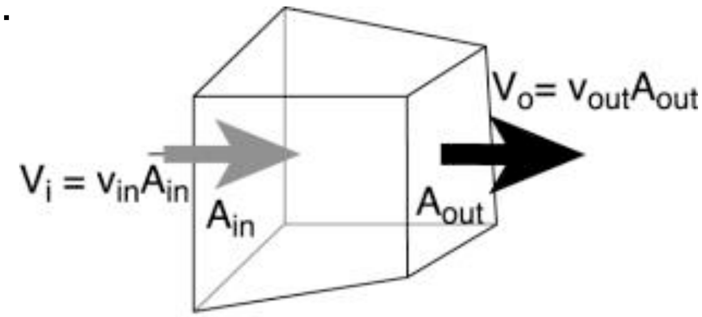
Displacement (x,y,z)
Velocity (u,v,w)

(x,y,z) = (0,0,0) is at local center of problem

Conservation of volume: Continuity at a point

Conservation at a point in the fluid (shrink the box to a point):

$$\nabla \bullet \vec{u} = 0$$



DPO Fig. 5.2

- 1D: $0 = \Delta u/\Delta x = \partial u/\partial x$
- 2D: $0 = \Delta u/\Delta x + \Delta v/\Delta y = \partial u/\partial x + \partial v/\partial y$
- 3D: $0 = \Delta u/\Delta x + \Delta v/\Delta y + \Delta w/\Delta z = \partial u/\partial x + \partial v/\partial y + \partial w/\partial z$
- (Net convergence or divergence within the ocean results in mounding or lowering of sea surface, or within isopycnal layers, same thing) NO holes in the ocean

- Note: the Δ's in numerator of each term refer to the change in value in the direction of the Δ in the denominator in that term.

Conservation of volume: continuity

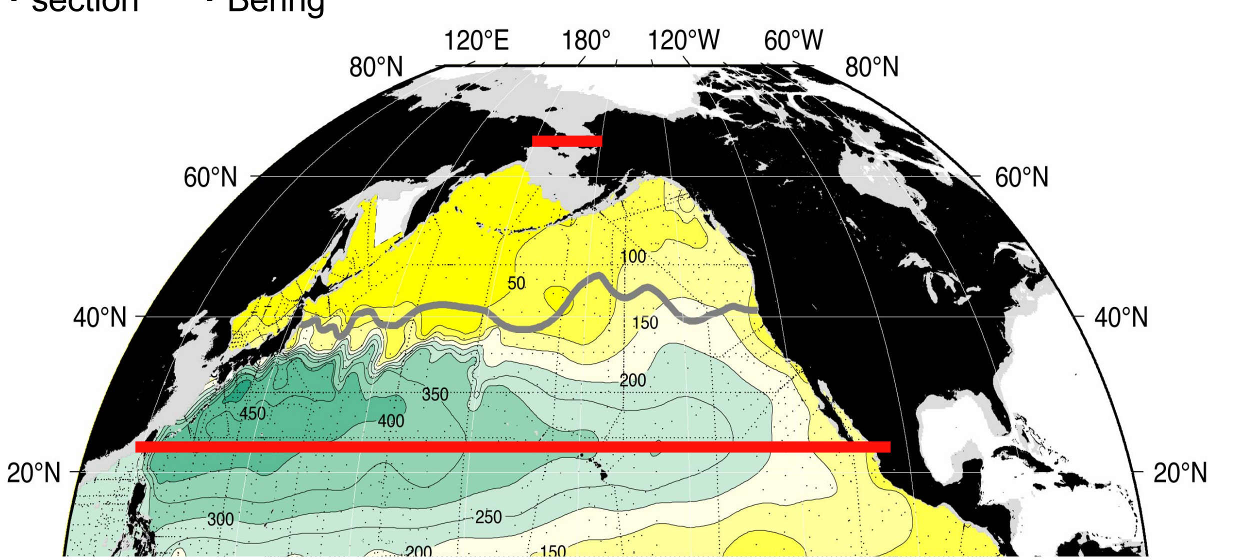
Example: for entire North Pacific (ignoring rain/evaporation*)

- The total volume transport $V_{section}$ in the north/south (meridional) direction across a coast-to-coast vertical cross-section (extending top to bottom)

EQUALS

- The total volume transport V_{Bering} through Bering Strait

$V_{section} = V_{Bering}$



Conservation of salt (steady state): salt transport

There is no external source of salt.

In steady state, which means the salt content doesn't change:

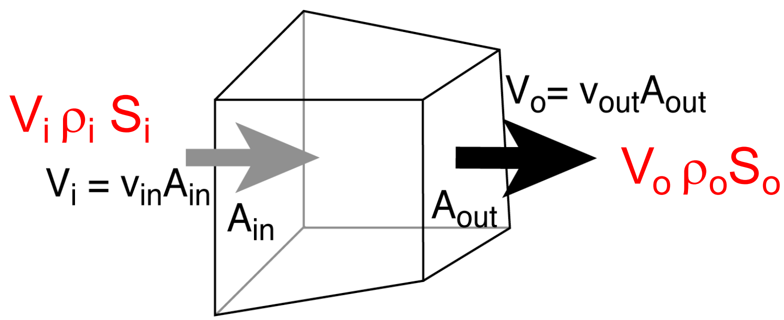
$$V_i \rho_i S_i = V_o \rho_o S_o$$

V = volume transport

ρ = density

S = salinity (expressed in kg/kg, not in g/kg!)

Subscripts "i" and "o" are "in" and "out"



DPO Fig. 5.2

Conservation of mass including rain, evaporation, runoff

$$F \equiv -\rho_o V_o + \rho_i V_i = -(R + A_s P) + A_s E$$

F = "freshwater" = net amount of precipitation, evaporation, runoff

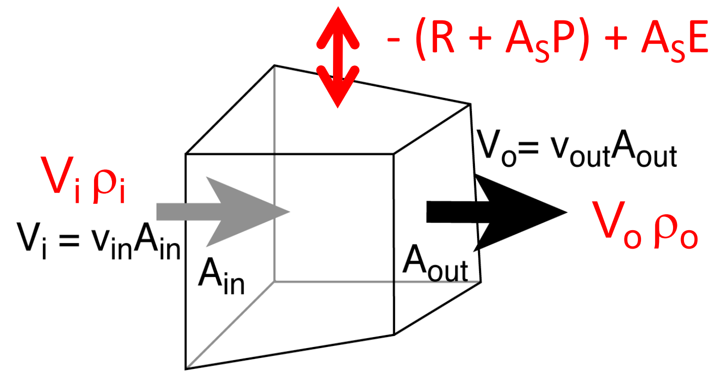
V = volume transport

A_s = surface area

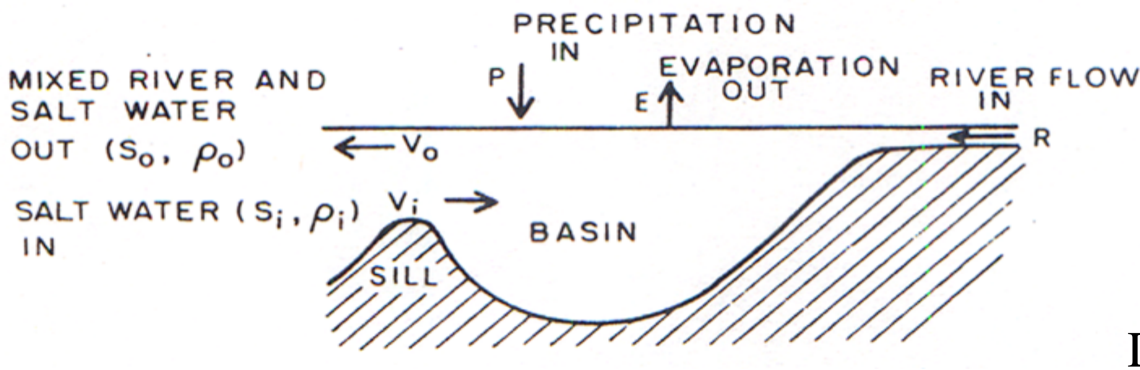
R = runoff

P = precipitation

E = evaporation



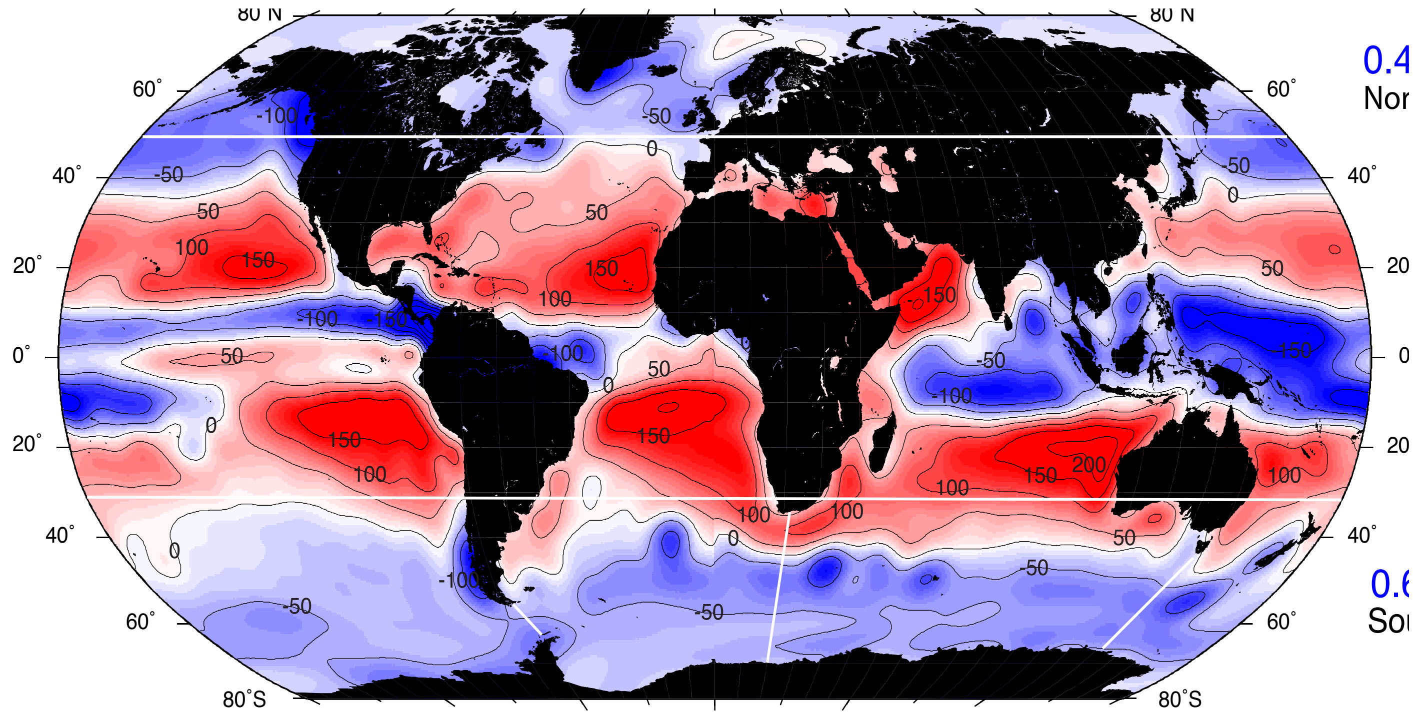
DPO Fig. 5.2



DPO Fig. 5.1

Surface freshwater source: Precipitation + runoff minus evaporation (cm/yr)

Annual mean: sustained by ocean transports (advection) that move fresher/saltier water



Net precipitation

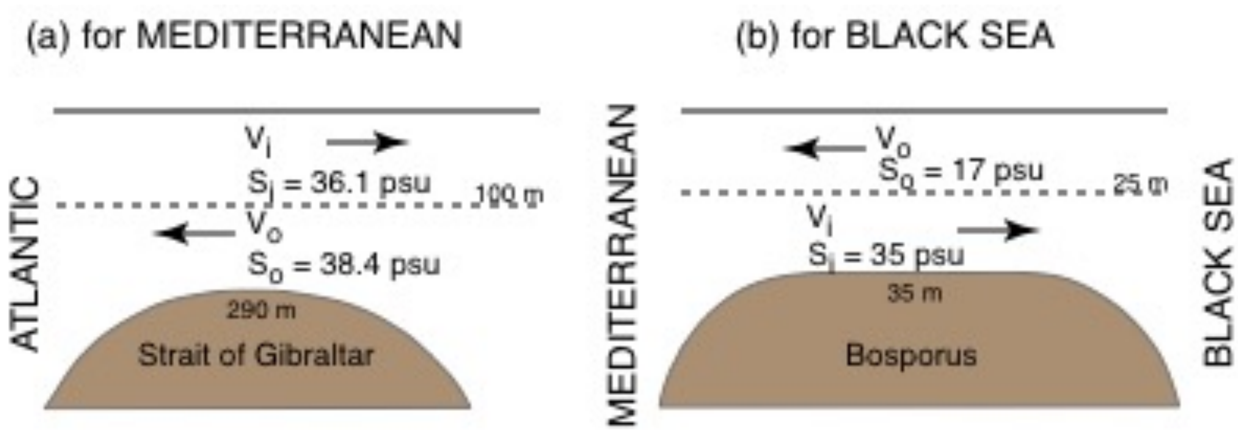
Net evaporation

Salinity is set by freshwater inputs and exports since the total amount of salt in the ocean is constant, except on the longest geological timescales

NCEP climatology DPO 5.4

Freshwater transport (steady state): Mediterranean and Black Seas

Flow in at top.
Evaporates and cools inside Med.
Water becomes denser.
Flow out at bottom.

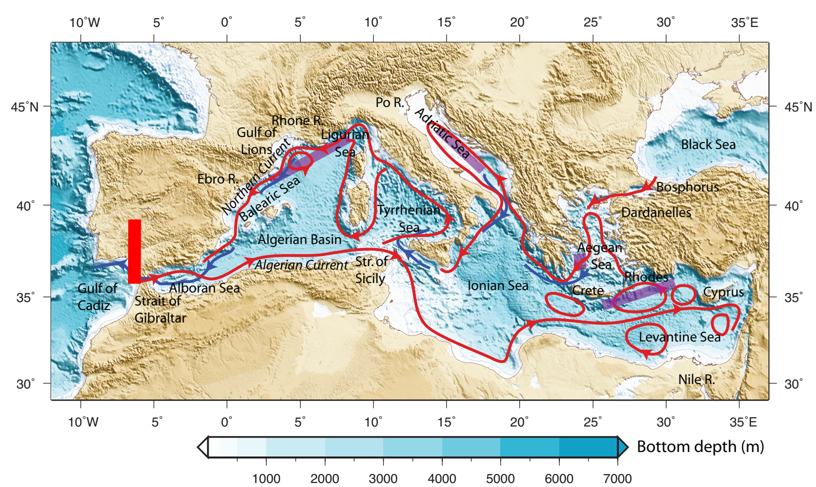


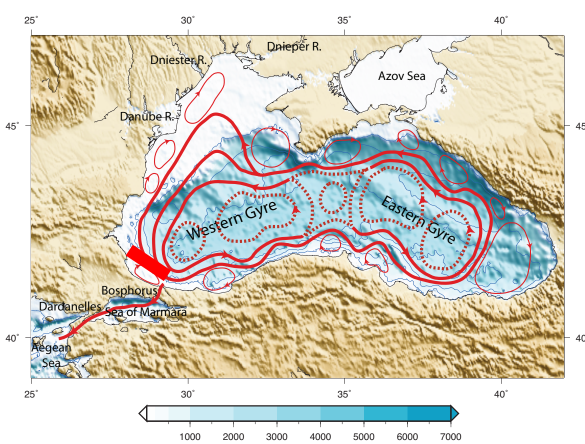
Flow in at botom.
Net runoff into Black Sea.
Water becomes lighter.
Flow out at top.

DPO Figure 5.3

Evaporative basin

Runoff/precipitation





DPO Figure S8.19 and S8.23

Conservation of freshwater: a practical approach

Mass: $F = -\rho_o V_o + \rho_i V_i = -(R + A_s P) + A_s E$ (positive is in to volume) (DPO defines positive as out of the volume)

Salt: $\xi = -\rho_o V_o S_o + \rho_i V_i S_i = 0$

Salt divided by an arbitrary constant, choose to be about equal to mean salinity S_m:

$$\xi / S_m = \rho_o V_o S_o / S_m - \rho_i V_i S_i / S_m = 0$$

Subtract $F - \xi / S_m = F - 0$

$$F - \xi / S_m = \rho_i V_i (1 - S_i / S_m) - \rho_o V_o (1 - S_o / S_m)$$

Assume $\rho_i V_i \sim \rho_o V_o = \rho V$ given how small F really is, so

$$\boxed{F \sim \rho V (S_o / S_m - S_i / S_m) = \rho V (S_o - S_i) / S_m = -(R + A_s P) + A_s E}$$

Remember this formula for Freshwater transport.

→ Freshwater input calculated from the difference in salinity between inflow and outflow equals the net precipitation, evaporation, runoff

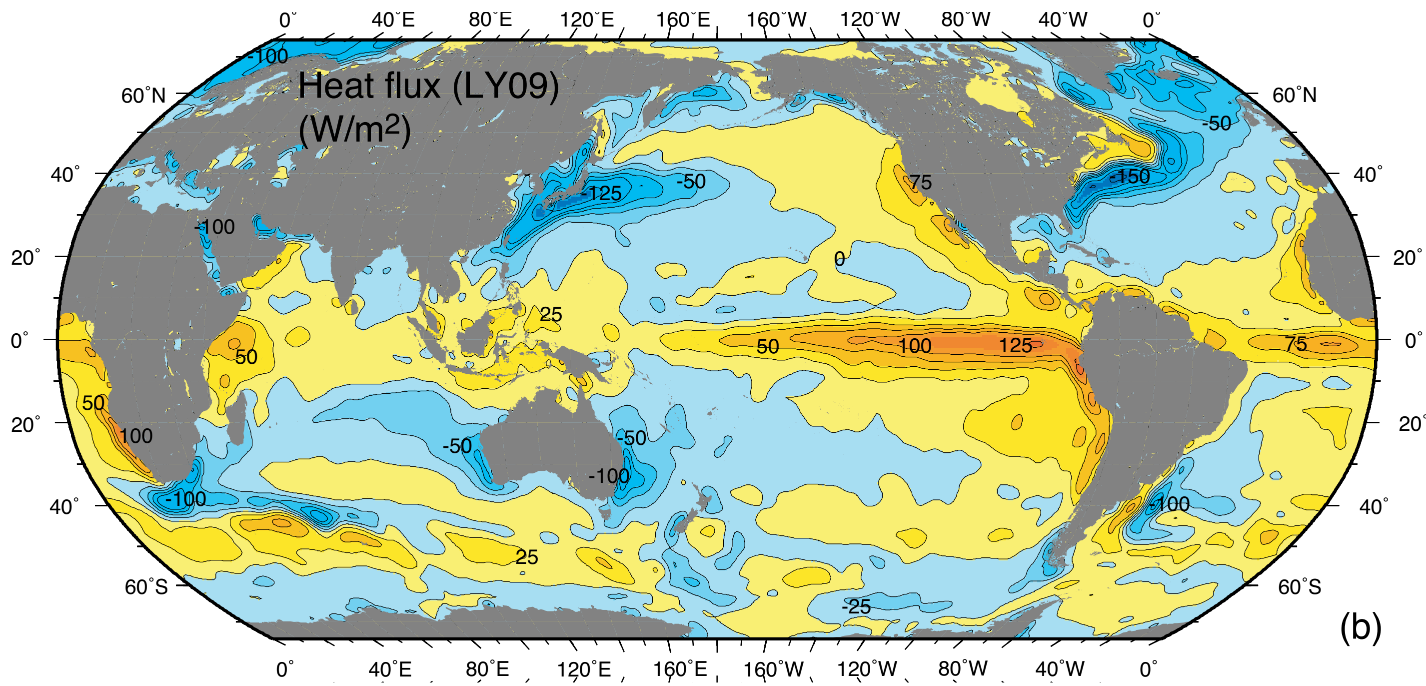
Heat and heat transport

Surface heat flux (W/m^2) Q into ocean (positive = ocean heating)

This map is the mean, annual surface heat flux. We can assume this is steady state.

How can there be continual net heat loss or net heat gain in different places?

Because there is transport (advection) of heat by ocean currents (and the atmosphere).

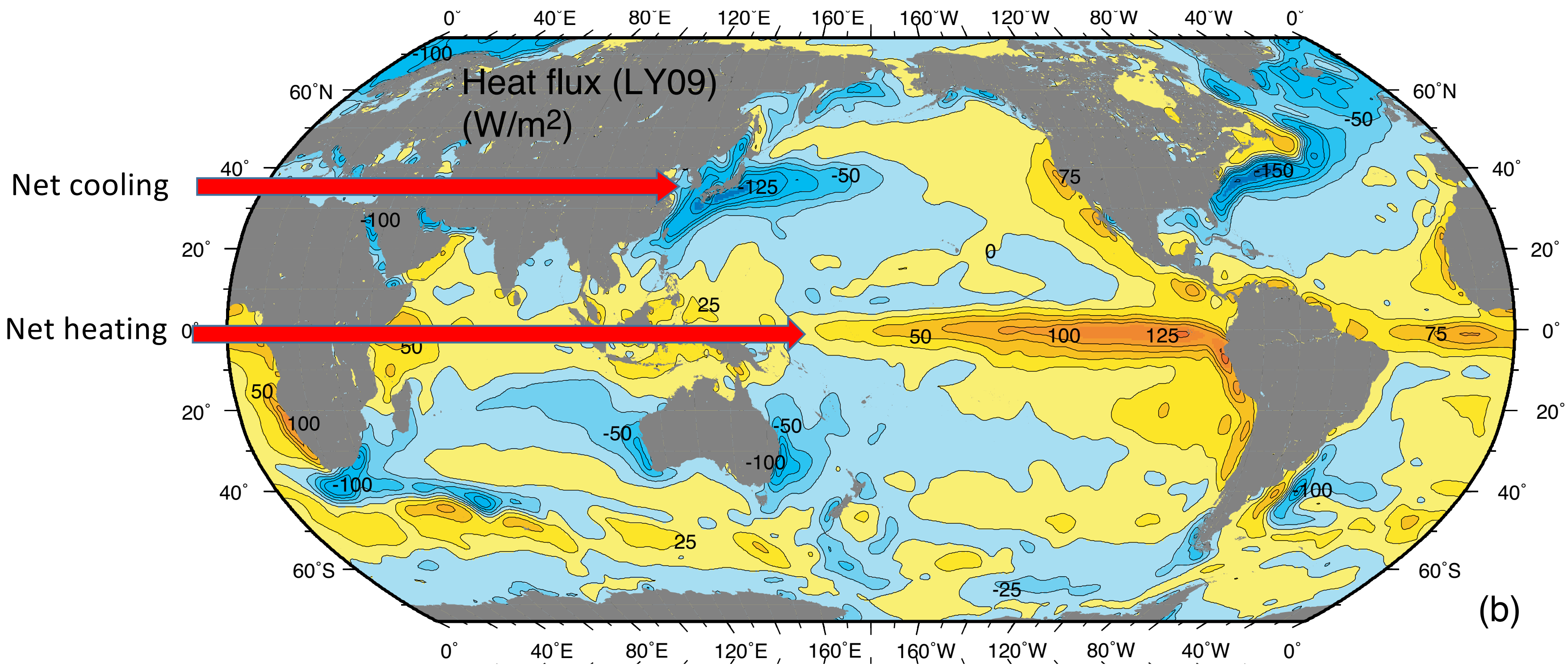


DPO Figure S5.8 (in supplement to Chapter 5)

Heat and heat transport

Surface heat flux (W/m^2) Q into ocean (positive = ocean heating)

This map is the mean, annual surface heat flux. We can assume this is steady state.
How can there be continual net heat loss or net heat gain in different places? Because there is advection of heat.



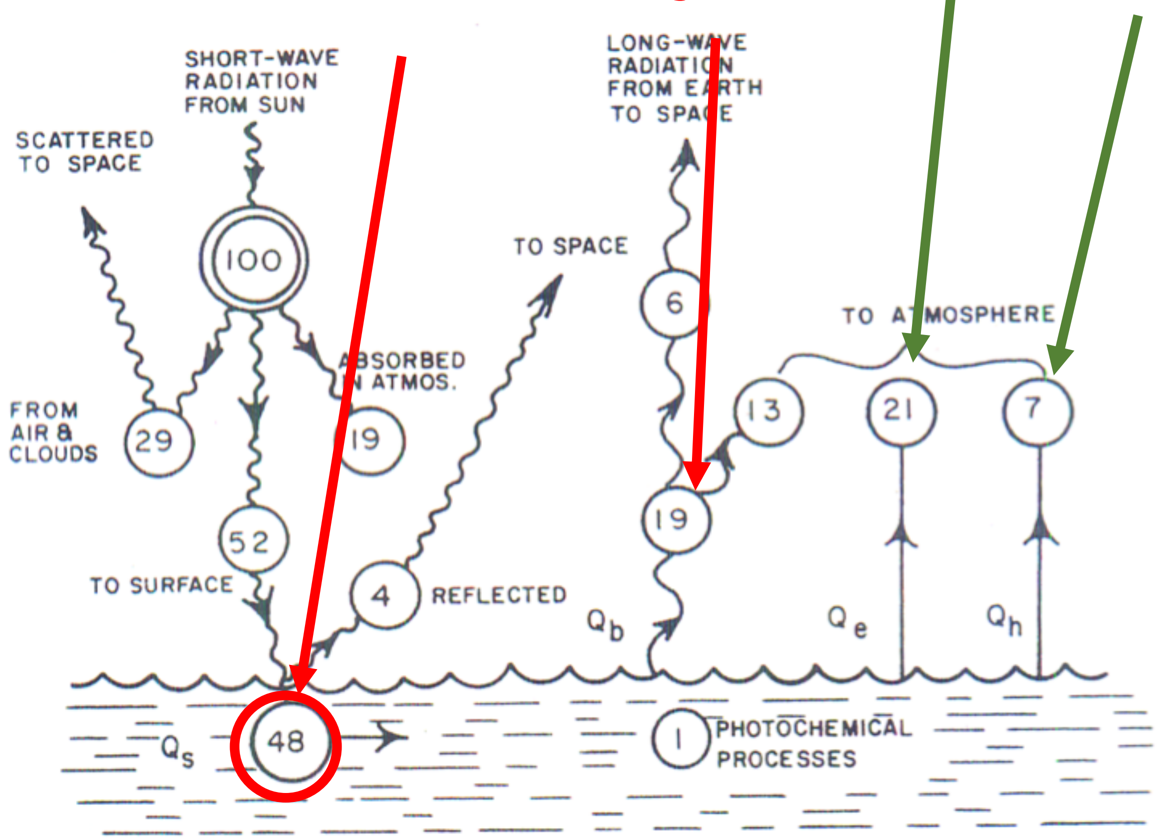
DPO Figure S5.8 (in supplement to Chapter 5)

Ocean surface heat flux contributions

$$Q_{sfc} = Q_s + Q_b + Q_e + Q_h$$

Total surface heat flux = Radiative terms + Turbulent terms

= Shortwave + Longwave + Latent + Sensible



This diagram shows a net global balance, not a local balance

DPO Fig. 5.5

Ocean heat balance

$$Q_{sfc} = Q_s + Q_b + Q_e + Q_h \text{ in W/m}^2$$

Shortwave Q_s: incoming solar radiation - always warms. Some solar radiation is reflected. The total amount that reaches the ocean surface is

$$Q_s = (1-\alpha)Q_{incoming}$$

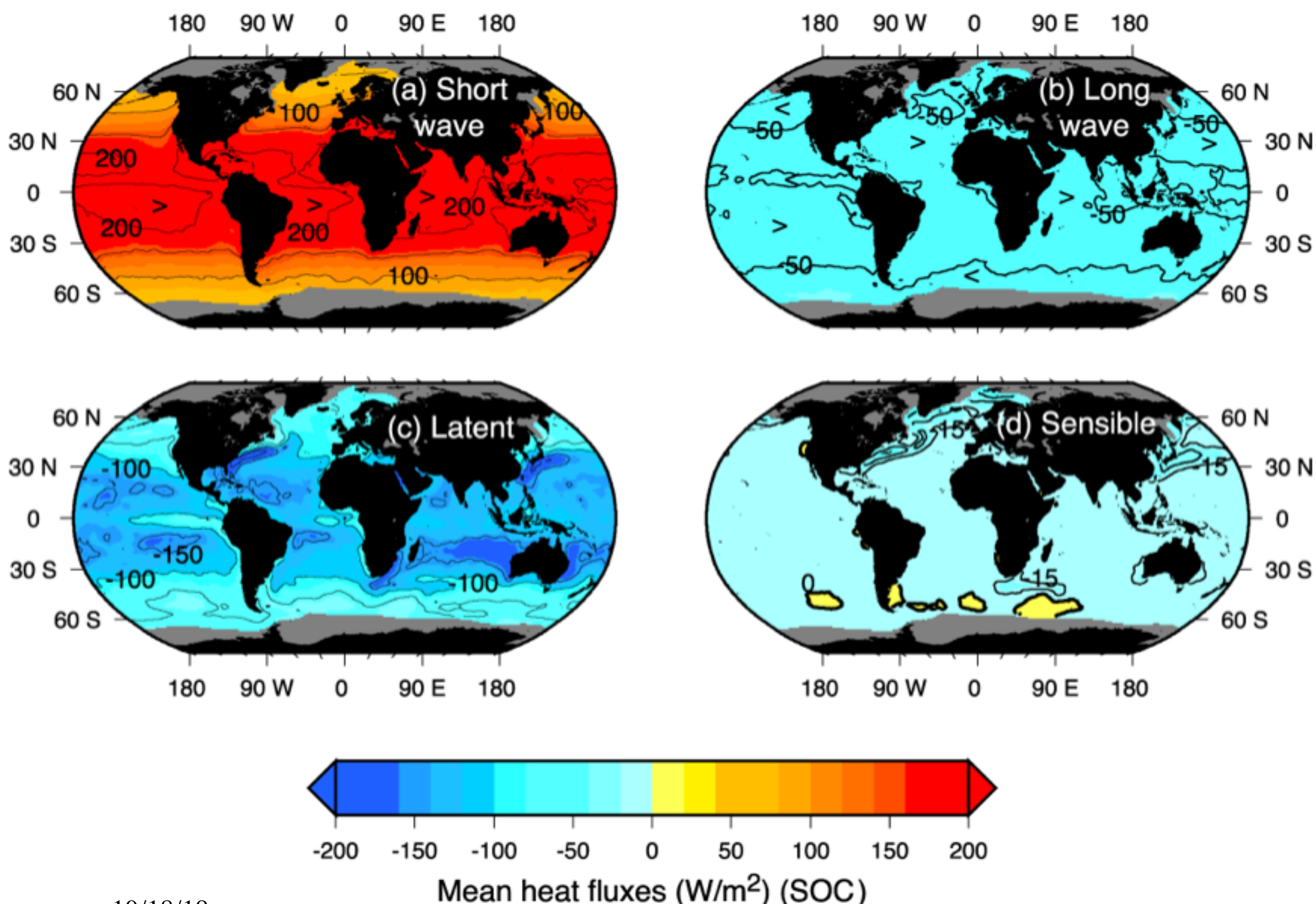
where α is the albedo (fraction that is reflected). Albedo is low for water, high for ice and snow.

Longwave Q_b: outgoing ("back") infrared thermal radiation (the ocean acts nearly like a black body) - always cools the ocean

Latent Q_e: turbulent heat loss due to evaporation - cools. It takes energy to evaporate water. This energy comes from the surface water itself. (Same as principle of sprinkling yourself with water on a hot day - evaporation of the water removes heat from your skin)

Sensible Q_h : turbulent heat exchange due to difference in temperature between air and water. Can heat or cool. Usually small except in major winter storms.

Annual average heat flux components (W/m²)



Radiative fluxes

Turbulent fluxes

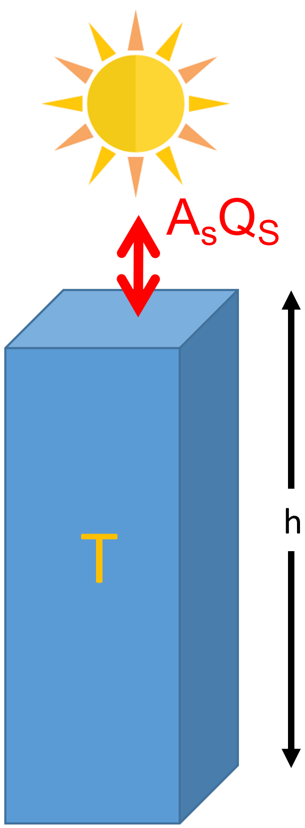
DPO Figure 5.11

a) Temperature change due to surface heat flux (time dependent, no transport/advection, no diffusi

Conservation of heat: time dependent (no advection or diffusion for this calculation)

$$d(H^* h A_S)/dt = d(\rho c_p T^* h A_S)/dt = A_s Q_S$$

$$\Delta T = [Q_S/(\rho c_p h)]\,\Delta t$$



T = temperature

t = time

H = Heat content per unit volume; h = height; Q_S = surface heat flux

Note that Volume = h * A_s so you can calculate change in T without assuming an area, just need thickness h of affected water column

Footnote – not necessary to learn this, but if you want to see where this comes from:
This is a special case of the temperature equation at end of next lecture.

$\Delta T/\Delta t + u(\Delta T/\Delta x) + v(\Delta T/\Delta y) + w(\Delta T/\Delta z) = (Q_S/h)/(\rho c_p)$ + diffusive terms

or

$\partial T/\partial t + u\,\partial T/\partial x + v\,\partial T/\partial y + w\,\partial T/\partial z = (Q_S/h)/(\rho c_p) + \partial/\partial x(\kappa_H \partial T/\partial x) + \partial/\partial y(\kappa_H \partial T/\partial y) + \partial/\partial z(\kappa_V \partial T/\partial z)$

Assume no advection, non-diffusive, uniform surface heat flux. Integrate over the volume.
Also assume that $Q(z) = Q_S\delta(z-z_o)$ where δ is the delta function.

a) Temperature change due to surface heat flux (time dependent, no transport/advection, no diffusi

Conservation of heat: time dependent (no advection or diffusion for this calculation)

$$d(H^*hA_S)/dt = d(\rho c_p T^*hA_S)/dt = A_s Q_S$$

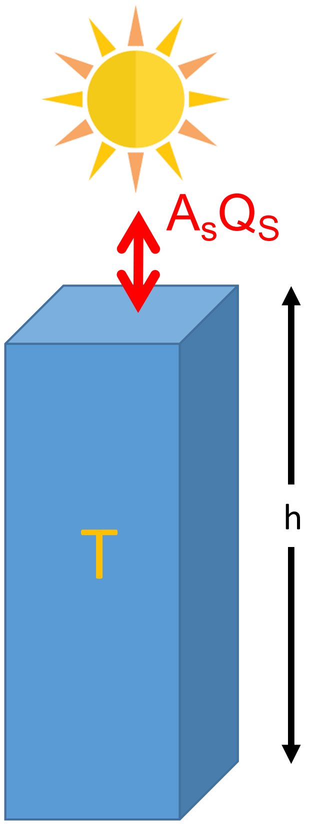
$$\Delta T = [Q_S/(\rho c_p h)]\,\Delta t$$

T = temperature

t = time

H = Heat; h = height; Q = heat flux

Note that Volume = height * A_s so you can calculate change in T without assuming an area, just need thickness h of affected water column



Compute the temperature change over 1 month in a volume of fluid that is 100 m thick, if the heat flux Q through the sea surface is 150 W/m^2. Remember that 1 W = 1 J/sec.

dT = (30 days* 24hr/day*60min/hr*60sec/min)(150 W/m^2)/(1025 kg/m^3 * 4000 J/kg° C * 100 m)

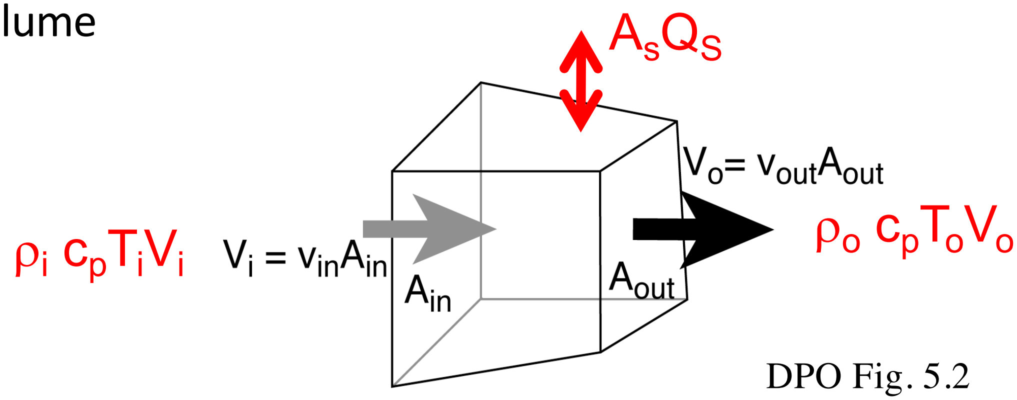
= 0.96 ° C or ~ 1° C

(b) Heat transport: how to balance mean surface fluxes

Conservation of heat including external sources, steady state, advection, no diffusion

$$H \equiv \rho_o c_p T_o V_o - \rho_i c_p T_i V_i = A_s Q_S$$

H = net heat flux (integrated) into the volume
T = temperature
c_p = specific heat
ρ = density
V = volume transport
A_s = surface area
Q_S = surface heat flux in W/m²



DPO Fig. 5.2

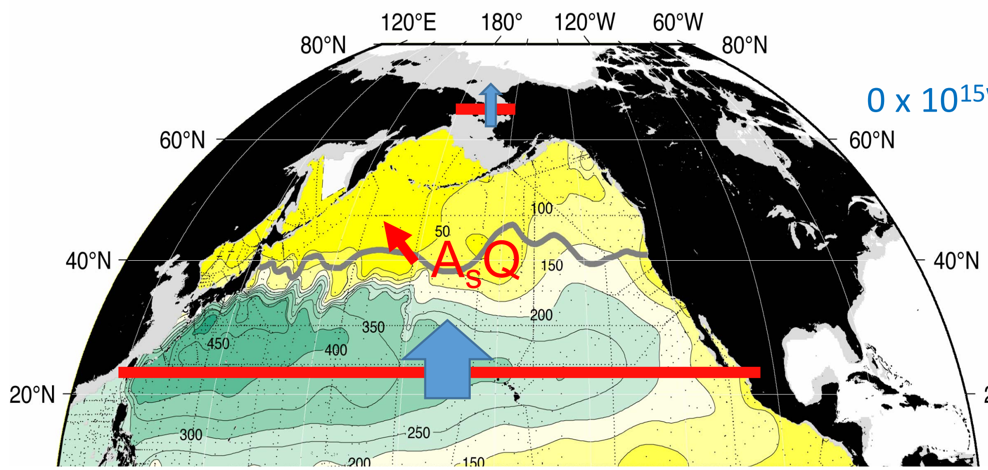
Footnote:
This is a special case of the temperature equation at end of next lecture.
$\Delta T/\Delta t + u(\Delta T/\Delta x) + v(\Delta T/\Delta y) + w(\Delta T/\Delta z) = (Q_S/h)/(\rho c_p)$ + diffusive terms
or
$\partial T/\partial t + u\,\partial T/\partial x + v\,\partial T/\partial y + w\,\partial T/\partial z = (Q_S/h)/(\rho c_p) + \partial/\partial x(\kappa_H \partial T/\partial x) + \partial/\partial y(\kappa_H \partial T/\partial y) + \partial/\partial z(\kappa_V \partial T/\partial z)$
Assume steady state, non-diffusive, uniform velocity over each side and uniform surface heat flux. Integrate over the volume. The remaining terms are horizontal advection (transport) and surface heat flux.

b) Conservation of heat including external sources, steady state, advection, no diffusi

$$\text{Heat} = H = H_o - H_i = \rho_o c_p T_o V_o - \rho_i c_p T_i V_i = A_s Q_S$$

Application: If northward heat transport across 24°N is 1.0×10^{15} W and heat transport across Bering Strait is 0W both relative to 0°C, what is the average surface heat flux between 24°N and Bering Strait? (make a reasonable estimate of surface area)

Arrow points UP (vertical)
A_S = surface area
Q = heat flux

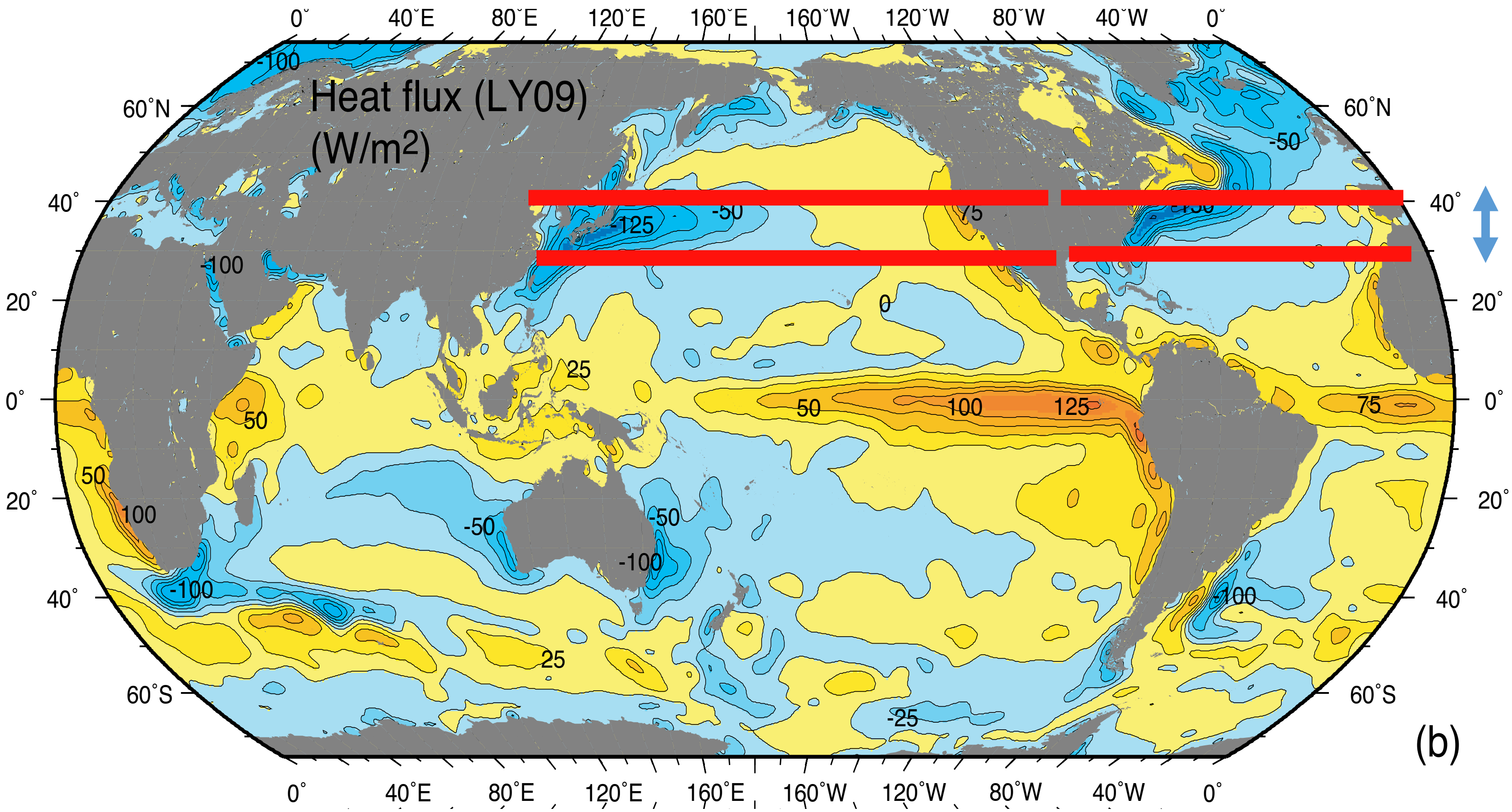


0×10^{15}W

1.0×10^{15}W

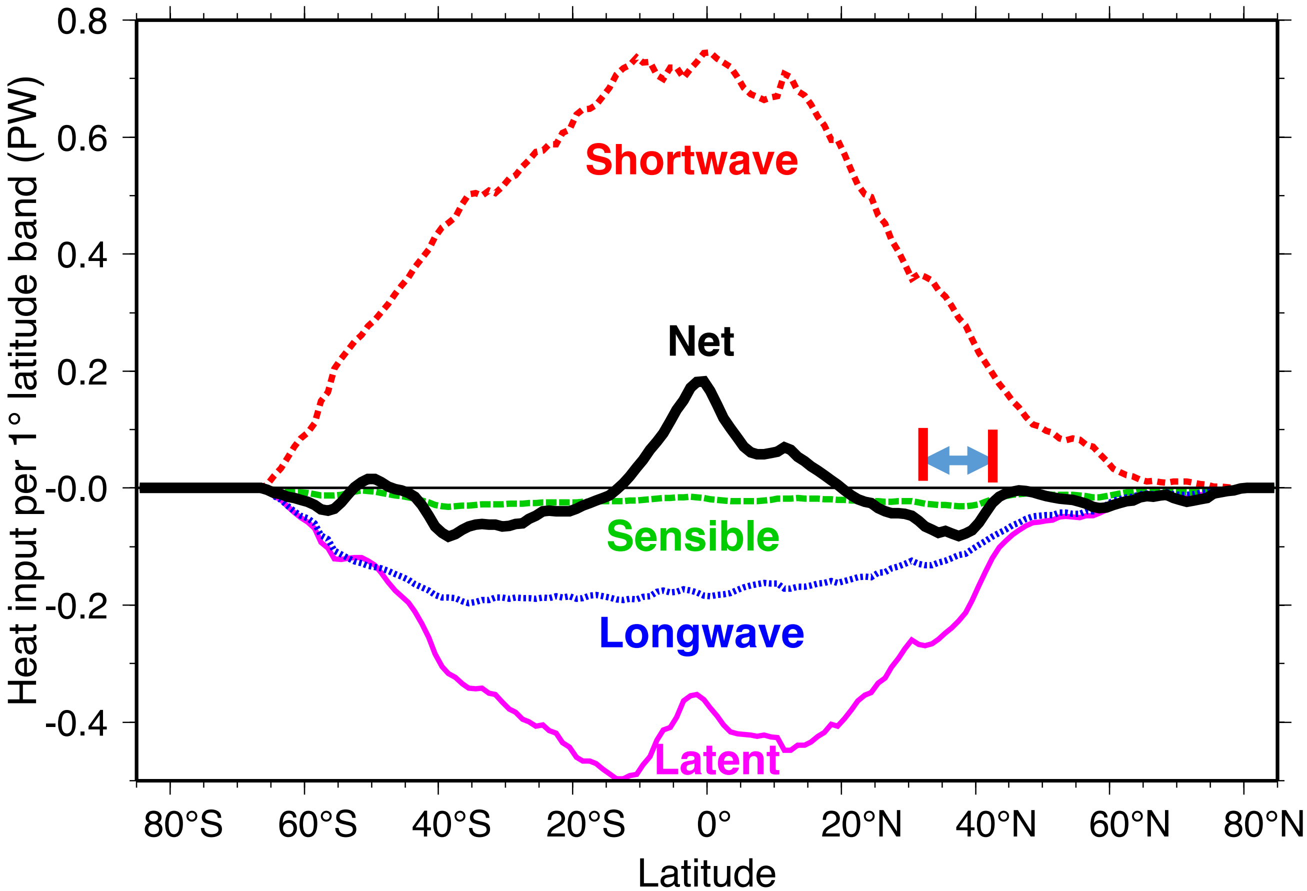
Sidenote: The cooled water does not all go out through Bering Strait. Where does it go? (subducts and moves southward back across 24°N)

Ocean heat budget and transports
Net surface heat flux (W/m^2) into ocean



DPO Figure 5.12

Heat flux components summed for latitude bands (W/m^2)

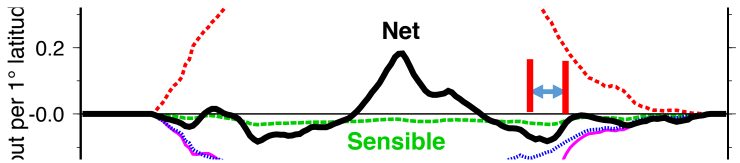


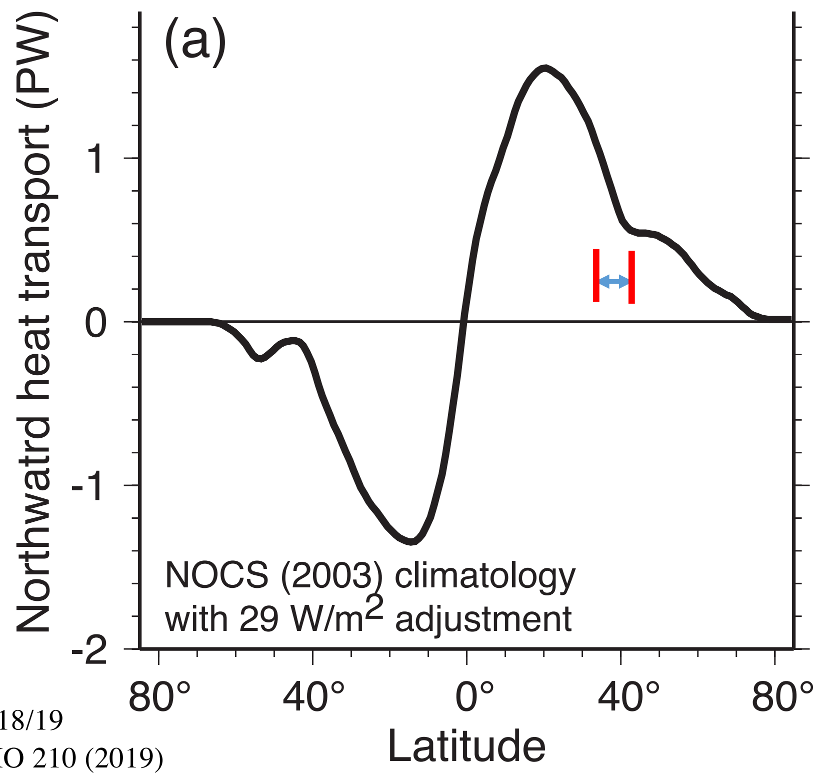
Heat input per latitude band (PW)

1 PW = 1 "Petawatt" = 10^{15} W

DPO Figure 5.13

Heat transport





Heat input per latitude band (PW)

1 PW = 1 “Petawatt” = 10^{15} W

Heat transport (PW)

(meridional integral of ‘Net’ in panel above).

e.g. at 30N, heat transport is northward and decreasing (transporting heat northwards from tropics and losing heat to the atmosphere)

DPO Figure 5.14