CHAPTER 1

Preliminaries

1.1 Geophysical Fluid Dynamics

The atmosphere and the ocean have so many fluid-dynamical properties in common that the study of one often enriches our understanding of the other. Experience has also shown that the recognition of the underlying dynamical concepts applicable to both the atmosphere and the oceans is an excellent starting point for the study of either. Geophysical fluid dynamics is the subject whose concerns are the fundamental dynamical concepts essential to an understanding of the atmosphere and the oceans. In principle, though, geophysical fluid dynamics deals with all naturally occurring fluid motions. Such motions are present on an enormous range of spatial and temporal scales, from the ephemeral flutter of the softest breeze to the massive and persistent oceanic and atmospheric current systems. Indeed, even the "solid" earth itself undergoes a fluidlike internal circulation on time scales of millions of years, the surface expression of which is sea-floor spreading and continental drift. All these phenomena can properly be included within the domain of geophysical fluid dynamics. Partly for historical reasons, however, the subject has tended to focus on the dynamics of large-scale phenomena in the atmosphere and the oceans. It is on large scales that the common character of atmospheric and oceanic dynamics is most evident, while at the same time the majestic nature of currents like the Gulf Stream in the ocean and the atmospheric jet stream makes such a focus of attention emotionally compelling and satisfying. This limitation will be observed in the following discussion, which consequently provides an introductory rather than exhaustive treatment of the subject. In particular the present text does not discuss the observational and descriptive features of meteorology and oceanography, although a familiarity with such evidence is a necessity for the proper formulation of new fluid-dynamical theories. Reference will be made from time to time in the text to the description of particular phenomena for the purpose of clarifying the motive for particular lines of study.

The principles to be derived are largely theoretical concepts which can be applied to an understanding of the natural phenomena. Such principles spring most naturally from the study of model problems whose goal is the development of conceptual comprehension rather than detailed simulation of the complete geophysical phenomenon. Geophysical fluid dynamics has historically progressed by the consideration of a study sequence within a hierarchy of increasingly complex models where each stage builds on the intuition developed by the precise analysis of simpler models.

1.2 The Rossby Number

The attribute "large scale" requires a more precise definition. A phenomenon whose characteristic length scale is fifty kilometers might be considered small scale in the atmosphere, while motions of just that scale in the oceans could be considered accurately as large scale. Whether a phenomenon is to be considered a large-scale one *dynamically* depends on more than its size.

For the purpose of this text large-scale motions are those which are significantly influenced by the earth's rotation. An important measure of the significance of rotation for a particular phenomenon is the Rossby number, which we define as follows. Let L be a characteristic length scale of the *motion*. Figure 1.2.1, for example, shows a typical wave pattern observed in the pressure field of the troposphere. A typical and appropriate length scale of the motion, i.e., one that characterizes the horizontal spatial variations of the dynamical fields, could be the distance between a pressure peak and a succeeding trough. Similarly let U be a horizontal velocity scale characteristic of the motion. In Figure 1.2.1 L would be O(1,000 km), while U would be O(20 m s^{-1}).*

The time it takes a fluid element moving with speed U to traverse the distance L is L/U. If that period of time is much less than the period of rotation of the earth, the fluid can scarcely sense the earth's rotation over the time scale of the motion. For rotation to be important, then, we anticipate

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* The symbol O( ) is used in two quite separate ways in this text. The statement that the functions $f(x)$ and $g(x)$ are in the relation $f(x) = O(g(x))$ (in some limit) implies that $f(x)/g(x) \rightarrow$ constant in that limit in a formal asymptotic sense. The symbol will also be used to mean that a variable quantity, in this case U, has a size exemplified by the value following the ordering symbol. No limit or approximation criterion is implied in the latter case. The two usages are distinct and the particular context will show clearly which is meant.

that

$$\frac{L}{U} \geq \Omega^{-1}, \tag{1.2.1}$$

or, equivalently,

$$\varepsilon = \frac{U}{2\Omega L} \leq 1. \tag{1.2.2}$$

The nondimensional parameter ε is the Rossby number. Large-scale flows are defined as those with sufficiently large L for ε to be order one or less. For the earth $\Omega = 7.3 \times 10^{-5}\ \mathrm{s}^{-1}$. For the L and U given above, $\varepsilon = 0.137$ and we can expect the earth's rotation to be important.

Such estimates must often be more refined. For planetary motions we shall see that it is really only the component of the planetary rotation perpendicular to the earth's surface which naturally enters the estimate of ε. Hence (1.2.2) could seriously underestimate the Rossby number for phenomena in low latitudes. Such elaborations and qualifications will be taken up later.

Note that the smaller the characteristic velocity is, the smaller L can be and yet still qualify for a large-scale flow. The Gulf Stream has velocities of order 100 cm s^{-1}. Although its characteristic horizontal scale as shown in Figure 1.2.2 is only O(100 km), the associated Rossby number is 0.07. Although the use of the local normal component of the earth's rotation would double this value at a latitude of 30°, it is still clear that such currents meet the criterion of large-scale motion.

Now these considerations have been essentially kinematic. However, the important dynamical consequence of even a moderately small Rossby number follows from the fact that small ε implies that large-scale motions are slow compared to the velocity imposed by the solid-body rotation of the earth. To a first approximation—i.e., to $O(\varepsilon)$—the atmosphere and oceans rotate with the planet with small but significant deviations which we, also rotating with the earth, identify as winds and currents. It is useful to recognize explicitly that the interesting motions are small departures from solid-body rotation by describing the motions in a rotating coordinate frame which kinematically eliminates the rigid rotation. In a frame rotating at a rate Ω only the deviations from solid-body rotation will be seen. Since such a rotating frame is an accelerating rather than an inertial frame, certain well-known "inertial forces" will be sensed, i.e., the centrifugal force and the subtle and important Coriolis force. We shall see that whenever the Rossby number is small, the Coriolis force is a dominant participant in the balance of forces. The study of the dynamics of large scale oceanic or atmospheric motions must include the Coriolis force to be geophysically relevant, and once the Coriolis force is included a host of subtle and fascinating dynamical phenomena are possible.

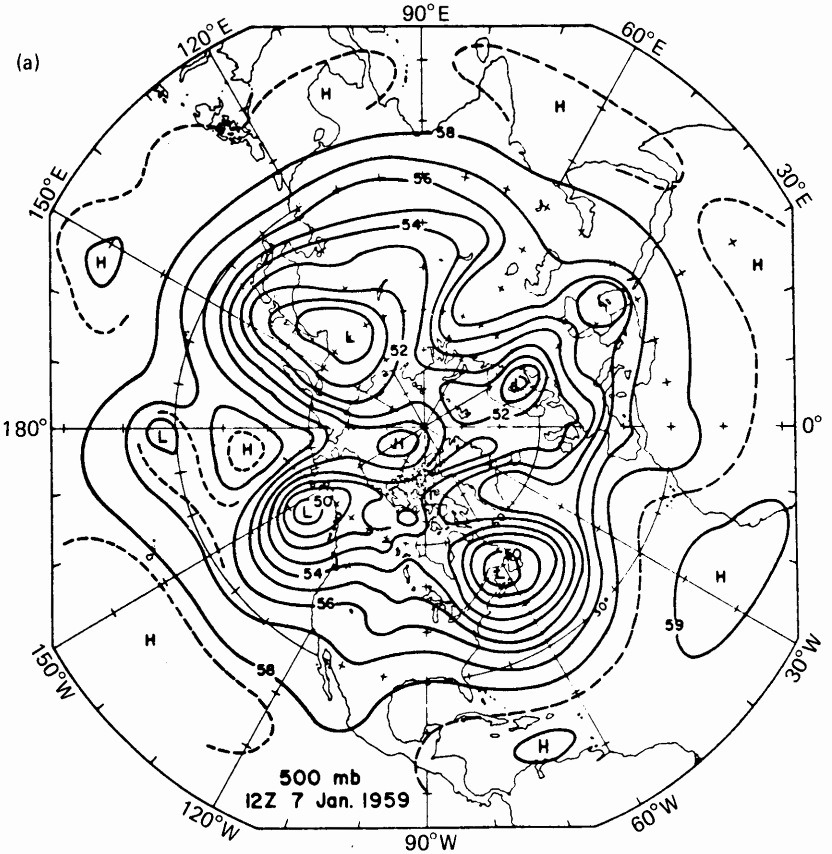


Figure 1.2.1(a) Isolines of constant pressure (isobars) at a level which is above roughly one-half the atmosphere's mass. The isobars very nearly mark the streamlines of the flow (Palmén and Newton, 1969).

(b)

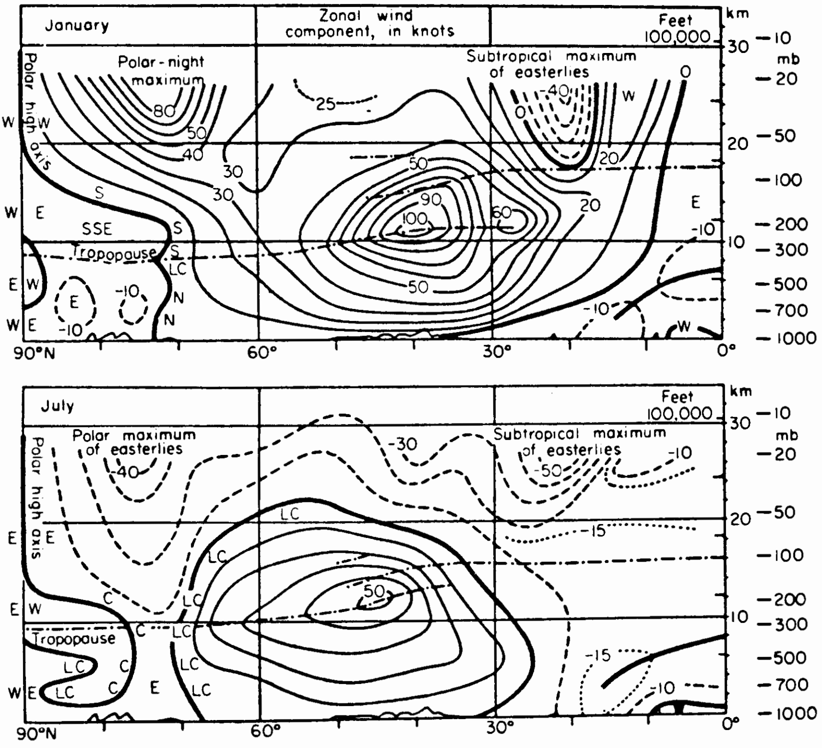
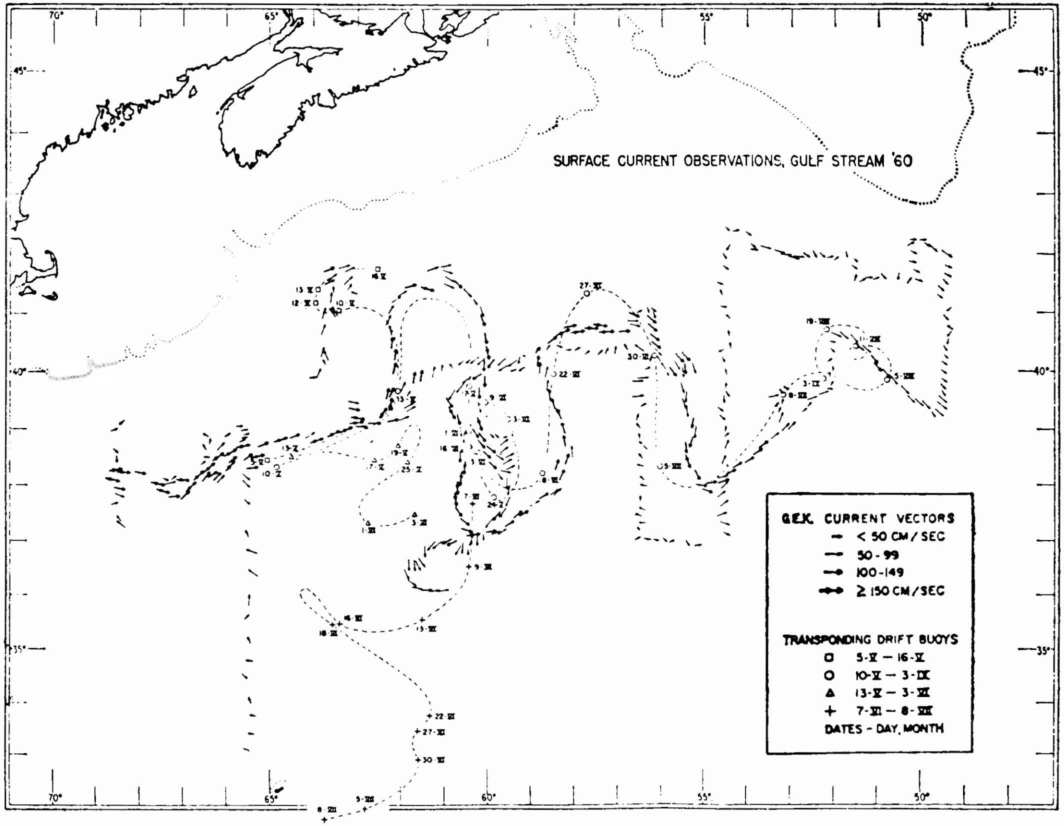


Figure 1.2.1(b) Cross section of the zonal wind (i.e., along latitude circles) showing the distribution of wind speed. (One knot ~ 50 cm s^{-1}) (Palmén and Newton, 1969, after Kochanski, 1955).


SURFACE CURRENT OBSERVATIONS, GULF STREAM '60
G.E.K. CURRENT VECTORS
< 50 CM/SEC
50-99
100-149
≥ 150 CM/SEC
TRANSPONDING DRIFT BUOYS
□ 5-V — 16-V
○ 10-V — 3-IX
△ 13-V — 3-VI
+ 7-VI — 8-VII
DATES - DAY, MONTH
70°
65°
60°
55°
50°
45°
40°
35°

(a)

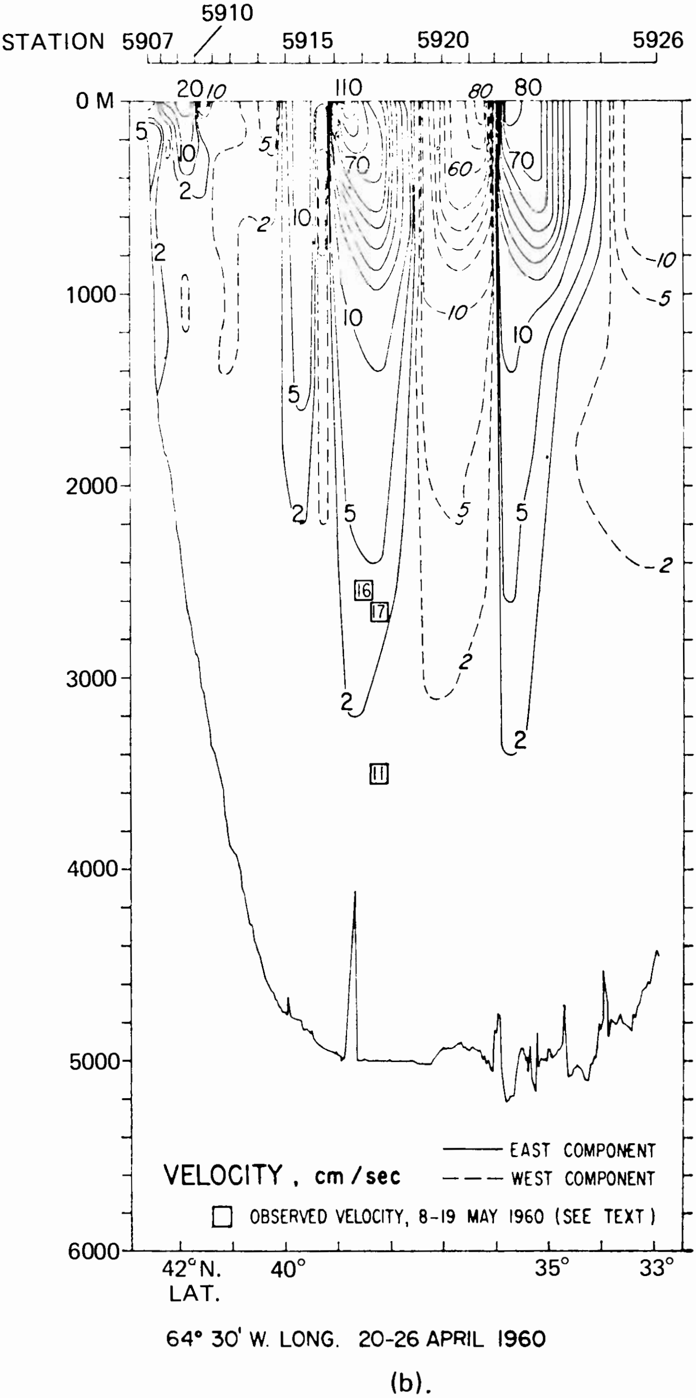


Figure 1.2.2 (a) (*Facing page*) The path of the Gulf Stream as revealed by surface observations, and **(b)** a cross section through the Stream which displays the structure of the current velocity (Fuglister 1963).