

210A (1)

Ocean Xpt processes

Stuff gets from one place to another by radiation, diffusion, advection

Stuff may be energy, momentum, heat, salt, nutrients, pollutants (not restricted yet!)

Xpt is quantified by idea of a flux magnitude, direction

Flux of stuff over area A = amt of stuff crossing 1 cm in direction \perp A in one sec.

$$[\text{Flux}] = [\text{stuff}] / \text{cm}^2 \text{sec}$$

Note Flux is in a direction, toward to A

Example: ① geothermal heat flux thru ocean floor = 10^{-6} cal/cm² sec

$$\text{Total heat out of ocean floor} = \text{Area} \times 10^{-6} \text{ cal/cm}^2 \text{sec}$$

$$= [\text{cal/sec}].$$

$$\textcircled{2} \text{ Top of cm } O^\circ \text{ soln efflux} = 1300 \text{ joule/m}^2 \text{ sec} = 1300 \text{ W/m}^2 \text{ cross from sun}$$

Radiation We usually think of EM radiation. It carries energy from the sun

+ returns energy to outer space. But EM transfer inside ocean is
weak because the ocean is not very transparent.

Energy/momentum transport by internal/surface gravity wave radiation
is imp. but defer its discussion until these waves have been described.
Seismologists concerned with energy Xpt by acoustic/near waves.

Diffusion ocean becomes mixed or always in motion thermal

motion even if material appears at rest.

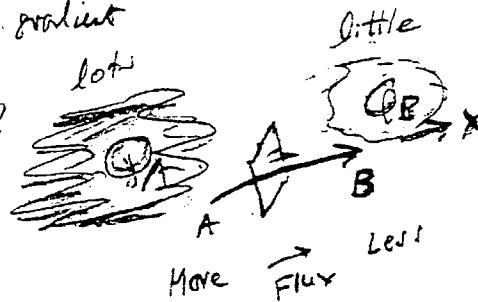
New concentration gradient is how stuff moves from somewhere else.

Described by Fick's law $Q_A = \text{stuff/cc at A} = \text{concentration}$

$$dQ/dx = \text{conc. gradient}$$

$$\vec{F} = -K \vec{\nabla} Q$$

$$\text{Flux A} \rightarrow B = -K \frac{(Q_B - Q_A)}{(B-A)} = -K \frac{dQ}{dx}$$



K is the diffusion constant

$$\frac{\text{stuff}}{\text{cm}^2 \text{sec}} = K \frac{\text{stuff/cm}^3}{\text{cm}}$$

$$[K] = \text{cm}^2/\text{sec}$$

$$\text{day} = 86400 \text{ sec}$$

$$m^2 = 3 \times 10^7 \text{ sec}$$

We have to measure K. If we know it we can answer some questions
How far (L) does stuff diffuse in time T? $L/T = K$ $L = \sqrt{KT}$

If stuff has diffused to L, how long (T) has it been diffusing? $T = L^2/K$

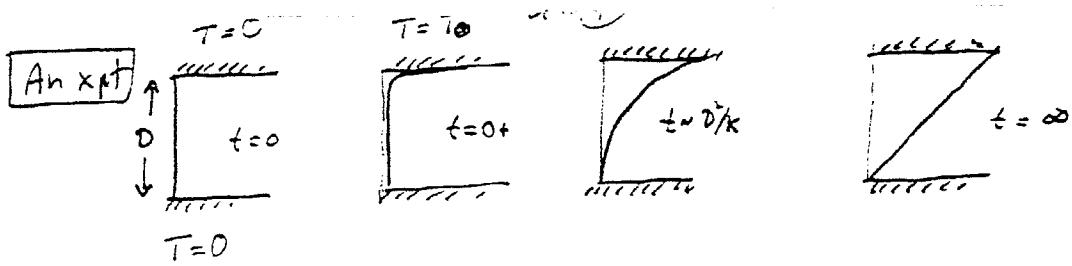
Example ① In arctic, surface temperature varies seasonally ($T = 86400 \times 180 = 10^7 \text{ sec}$,

Heat diffuses at $K = 10^{-7} \text{ cm}^2/\text{sec}$ in dirt. How deep (L)

is seasonal temp. change noticed? $L = (10^7 \times 10^{-7}) \text{ m} = 1 \text{ m}$.

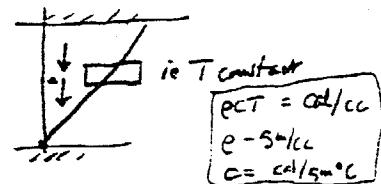
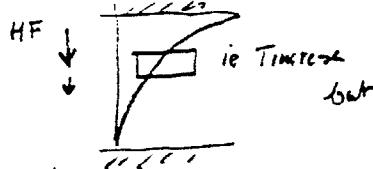
② How long must year be for seasonal temp var to penetrate 10 km?

$$T = L^2/K = (10^7)^2 / 10^{-7} = 10^9 \text{ sec} = \frac{10^9}{3600} \text{ years}$$



Notice

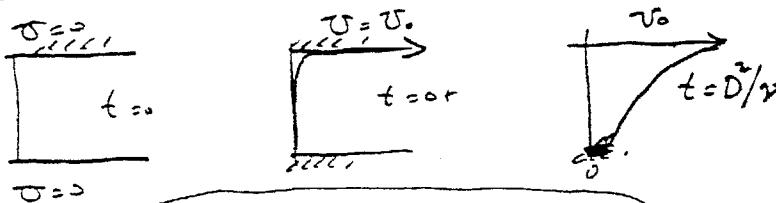
$$HF = -K \frac{\Delta H}{\Delta z} \\ H = \rho c T$$



$$\text{Heat Flux } \downarrow = -\frac{\Delta \text{HEAT conc}}{\Delta z} K = -K \rho c \left(\frac{T_0 - T_B}{D} \right)$$

Top plate heats bottom

Now another xpt superficially different but really the same. Momentum (x) diffuse



$$\text{mom/vel} = \rho \times \text{velocity} = \frac{8 \text{ gm cm/sec}}{\text{cm}^2} = \frac{5 \text{ m}}{\text{cm}^2 \text{ sec}}$$

$$\tau_{xz} = X \text{ Force / area} = -\nu \frac{\partial U}{\partial z}$$

$$\frac{\text{Force}}{\text{area}} = \frac{8 \text{ gm cm/sec}}{\text{cm}^2} = \frac{\text{cm}}{\text{sec}} \frac{\text{gm}}{\text{cm}^2} \frac{\text{cm}}{\text{sec}} \frac{1}{\text{cm}}$$

$$= \frac{8}{\text{cm sec}}$$

Top plate pulls bottom with force τ_{xz} per area

x momentum diffuse with coefficient ν :
 ν is the viscosity.

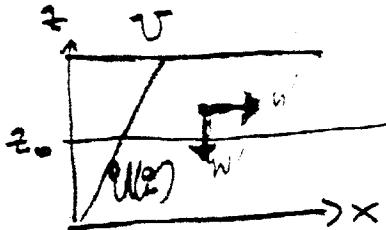
$\tau_{xz} < 0$ top pulls bottom to move film off

$$MF = -\nu \frac{\Delta M}{\Delta z} \quad M = e^{U(z)}$$

Molecular basis of momentum transport in a gas.

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Molecules have random thermal speed u' in all directions plus mean flow \bar{v} in $(\text{say}) x$ direction. Molecules go freely distance l (w.f.p.) between collisions. Molecule crossing z_0 from upper to lower layer cause extra mean momentum $\bar{v}(z > z_0)$ with it & deposit that in lower layer after collision thus upper layer drag lower layer drag.



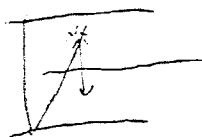
$$\text{i.e. } \overline{\tau}_{xz} = -\rho \underbrace{(u' l e)}_{2} \bar{v}$$

In gas, increasing T increases u' , $\bar{v} \uparrow$

In liquid - - - molecular separation, int. molecule force $\bar{v} \downarrow$.

Size of $v, K \dots$ in sea water

v	0°C	$.010 \text{ } 20^\circ\text{C}$	10^{-6}
K_T	.0014	cm ² /sec	1.4×10^{-3}
K_s	.000013	-	1.3×10^{-5}



$$\begin{aligned}
 \overline{\tau}_{xz} &= +\rho w' u' \left(\frac{\partial u'}{\partial z} \right)_0^0 \\
 &= \bar{v} \downarrow \uparrow \\
 &= +\rho \left[-u' \bar{v} (B_0 + B_1) + \frac{u'}{u} \bar{v} (B_0 - B_1) \right] \\
 &= -\rho \langle u' l e \rangle \bar{v}_2
 \end{aligned}$$

tracer when mass of fluid carries stuff

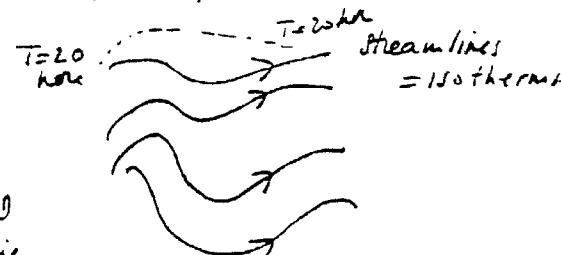
$$\text{Flux} = \overrightarrow{\text{velocity}} \times \text{concentration} (\text{mg/l}) \cdot [\text{stuff}/\text{m}^2]$$

so simple it seems useless to ponder.

Application In 2D iso-tracers are streamlines of steady flow

i.e. if flow were steady + 2D,

measuring one tracer + contouring
iso-lines would tell us streamline
(but not flow speed)



In ocean flow is often idealized
as steady but is really 3D. Streamline
lie anywhere on isobaric sheets.

Measure 2 tracers. Their sheets usually
aren't parallel but intersect.

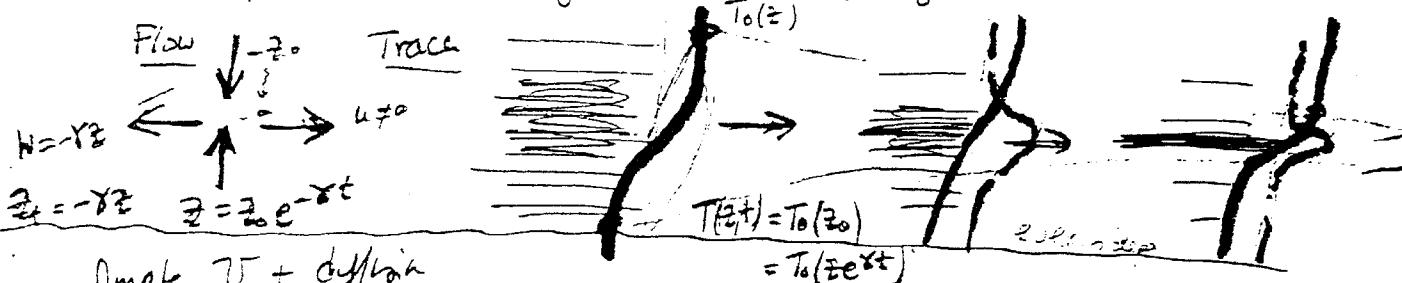
Flow lies in both sheets i.e.

SL's are intersections of sheets



Simple UT - another application

Note how convergence can sharpen gradients

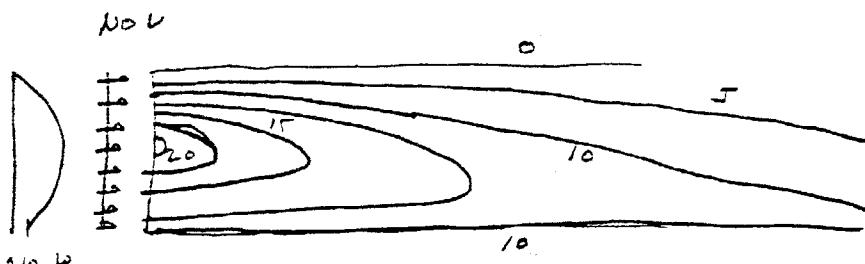
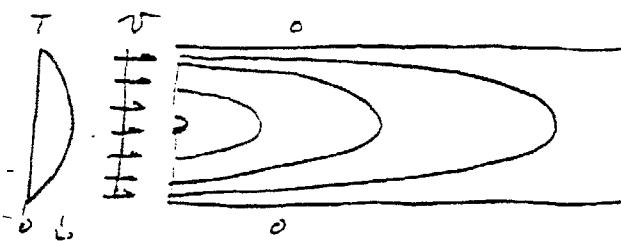


You sketch if $UT \rightarrow \infty$
or $K \rightarrow \infty$

Notice tongue $\Rightarrow \rightarrow$

N.B. Fluid film thru ice cream
surface of contact?

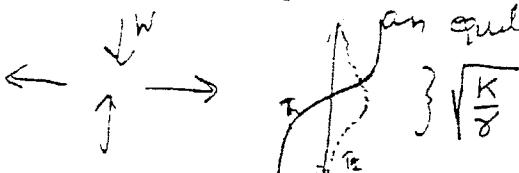
$N+P$ 40°
Principle of T_0



Now axis of tongue
bends down
NOT $\parallel UT$

Convergence + diffusion

Advection sharpens T gradient; diffusion smooths
an equilibrium can exist: $W = vT$

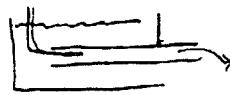


$$N.B. [\delta] = \frac{1}{t} - 1$$

= little like \sqrt{KT}

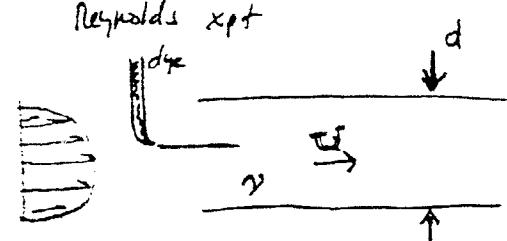
$$\begin{aligned} -vT_2 &< kT_{22} \\ \ln T_2 &= -\frac{vT_2}{k} \\ T_2 &= \text{const } e^{-vT_2/k} \\ &\propto UT \end{aligned}$$

What is \bar{U} like in stratosphere/ocean? Is it simple?



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Reynolds expt



If $\frac{Ud}{v}$ small enough flow is laminar
ie in sliding sheets. Dye not dispersed



$$\text{laminar} = \frac{\bar{U} \cdot d \cdot \nu}{\eta} \cdot \text{Re}$$

If $\frac{Ud}{v}$ too big flow is turb. Dye dispersed

Very $\frac{Ud}{v}$

$\text{Re} = \bar{U}d/\nu$ is ∞ . Rec't depth or geometry. In ocean \sim by temperature

typical $\text{Re} = (10^2 \text{ cm/sec}) (4 \times 10^5 \text{ m} = \text{depth}) / 10^{-1} = 4 \times 10^9$.

This is naive but still much of ocean is turb. Whether everywhere/always not clear - there may be nearly quiescent patches. How big???

Stretch/cap?

Lesson is that \bar{U} is probably very complicated.

In satellite images many short retrograde features $\leftarrow \rightarrow$

Complex \bar{U} + diffusion

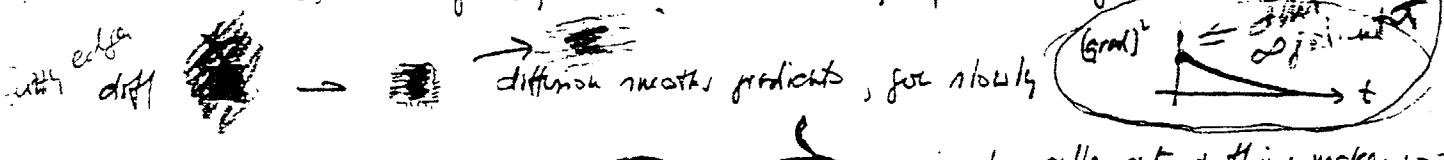
spot

Consider putting milk into coffee. (1) Eject in, let spread molecularly.

$$\text{Time to stir} = L^2/K = (0.01)^2 / 0.01 = 10^4 \text{ sec. !!!}$$

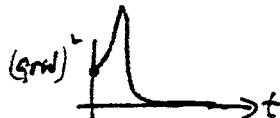
(2) let coffee to rotate smoothly. Spot just rotates

(3) stir vigorously. In c. second or so, complete mixing. Why?



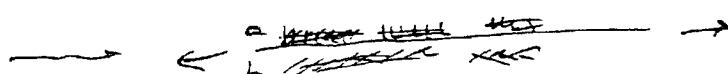
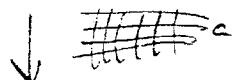
ie adv pulls out + thin makes + keeps gradient by diff for fast

Eckort:



people often say stirring + diffusion \approx big K diffusion.

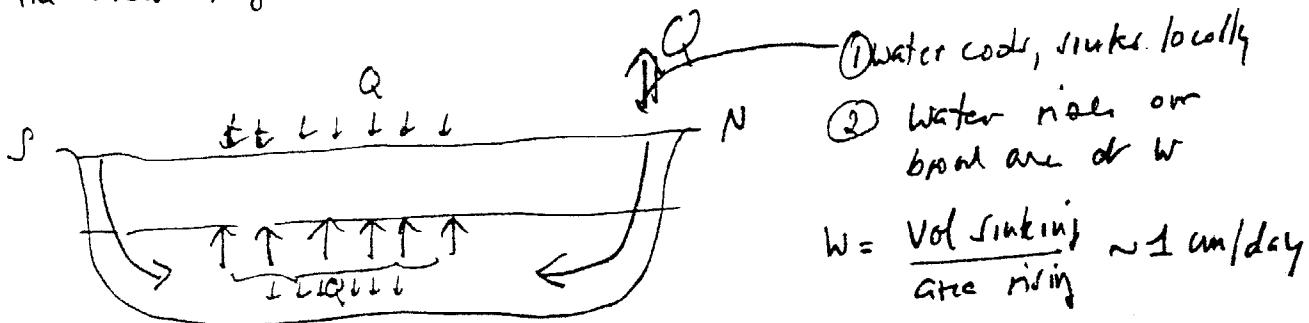
big K is eddy diffusivity. This is not true! Kelly



There are two attitudes in the literature

- (1) accept & use
- (2) look further

The accept - & - be procedure chosen Kelly to fit the answer right. Munk's original recipe is an example DSR 66 707-3T



$$W = \frac{\text{Vol sinking}}{\text{Area rising}} \approx 1 \text{ cm/day}$$

$$(J) \quad \frac{\Delta T}{z} \quad \left. \begin{array}{l} \uparrow \\ \downarrow \end{array} \right\} 100 \text{ m}$$

(4) Rising water carries heat up,
to keep ~~constant~~
~~different~~ constant deep water at constant
temp. must differ heat downward. HF same everywhere
 $HF_A = -K T_2 + WT \quad HF_B = 0$

~~constant~~
At initial latitude, temperature.

$$WT = K \frac{\Delta T}{\Delta z}$$

$$WT = KT_{22} \quad \frac{WT}{\Delta z} = K \frac{\Delta T}{\Delta z}$$

$$\Delta T = T$$

$$K \approx \frac{WT}{\Delta z} \approx 10^{-5} \frac{\text{cm}}{\text{s}} \times 10^5 \text{ cm} = 1 \frac{\text{cm}}{\text{s}}$$

Compare with $K_{\text{mol}} = 10^{-2} \text{ cm}^2/\text{s}$.

When we think 'back into' the answer we havn't really said why K_{mol} is what we get. Always $K_S > K_{\text{mol}}$. Typically

$$K_{\text{EV}} = 1 - 10^2 \text{ cm/sec} \quad K_{\text{SH}} = 10^4 - 10^8 \text{ cm/sec}$$

Part of this due to Depth \ll lateral extent of ocean.

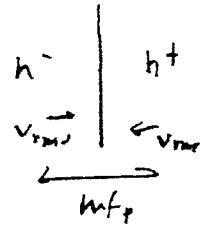
More due to vertical extent, rms w of chd \ll heat, T_{mol} .

That mainly due to stable stratification light fluid/heavy

How we 'prove' $WT = K \frac{\Delta T}{\Delta z}$ double + close?

People have tried to do ~~estimate~~ along the molecular line.
 Imagine ~~a~~ gas (air) with small concentration of some other gas of similar molecule - that concentration is $n(x,t)$. All molecules travel with thermal velocity v_{rms} between collision, distance between collision is mfp. Imagine n is space variable. Flux across surface is

$$\begin{aligned} \text{flux (solvent atoms/sec/cm}^2) &= n^- v_{rms} - n^+ v_{rms} \\ &= -v_{rms} (n^+ - n^-) \\ &= -(mfp) v_{rms} \frac{dn}{dx} \end{aligned}$$



i.e. diffusivity is $K = (mfp) v_{rms}$.

Could interpret $v_{rms} = v_{rms}^{fluid} + mfp = \text{eddy size}$

& say $K_{edd} = (\text{eddy size})(v_{rms}^{fluid})$

Engineering hope is that once K_{edd} is 'calibrated', can moderately predict.

I've said this vaguely, what is 'eddy size'. You can devise estimates of same length scale to put in, but then remain problems.

Yet the idea that small-scale flow features diffusion is useful
 Distinction between large/small = average, not flux anymore

~~One distinction between large & small flow features is that~~, usually;
~~it is that we can measure~~ a matter of estimation. In the ocean, our ability to observe small details asymptotically is limited. That almost every observation is really at \rightarrow a large scale that averages out small scale.

Thus if we try to measure S , we really set an extent of $\langle S \rangle$.
 We might want $\langle \text{advection flux} \rangle = \langle \bar{u}S \rangle$. (called Reynolds decomposition)

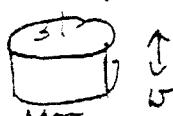
$$\text{But really } \begin{aligned} ① S &= \langle S \rangle + S' = \text{mean} + \text{rest} & \langle S' \rangle &= 0 \\ ② u &= \langle u \rangle + u' & \langle u' \rangle &= 0 \end{aligned}$$

$$\text{so that } ④ \langle uS \rangle = \langle \langle u \times S \rangle + \langle u \rangle S' + u' \langle S \rangle + u' S' \rangle \quad \text{Reynolds flux}$$

$$= \langle u \rangle \langle S \rangle + \langle u' S' \rangle$$

The advection flux has a part due to mean flow & part due to fluctuation. Then
 Remarkably, very often $| \text{Reynolds} | \gg \text{mean}$.

For example: boil water, let $\langle \rangle$ be over pan area.



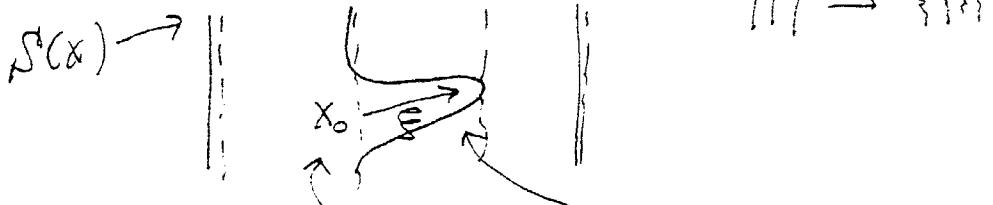
$$\begin{aligned} HF &= WT_c \quad \langle HF \rangle = \langle W \times T \rangle_c + \langle W' T' \rangle_c \\ \text{but } \langle W \rangle &\equiv 0!! \quad \langle W' T' \rangle_c = \langle W >_0 \times T >_0 \dots \\ &\quad + W < 0 > \times T < 0 > \dots = W_p - \\ &\quad = \langle \text{heat rises} + \text{condensates} \rangle \end{aligned}$$

Sometimes we can measure u' , s' . The eddy flux hypothesis says

$$\langle u' s' \rangle = -K_{edd} \frac{d\langle s \rangle}{dx}$$

How can this be?

Taylor: imagine a smooth initial salt field $s(x)$ + let small scale motion disrupt w/o ~~any~~ molecular diffusion



Poral starts at x_0
goes to $x_0 + \delta$

$$s'_{\text{there}} = s'_{\text{here}}$$

$$\begin{aligned} s'_{\text{here now}} &= s'_{\text{here now}} - s'_{\text{here then}} \\ &= s'(x_0) - s'(x_0 + \delta) \\ &\approx -\delta \frac{ds'(x)}{dx} \end{aligned}$$

Average $\langle \cdot \rangle$ over small scale displacement:

$$\boxed{\langle u' s' \rangle = -\langle u' \delta \rangle \frac{ds'}{dx}}$$

i.e. to set Keddy measure u' = local velocity
and δ 'recent' displacement, $K = \langle u' \delta \rangle$.
You could measure these quantities with fluid following drifters.

Note conceptual #'s — what does 'recent'
mean — yet approach is useful

Measuring u' and σ is not easy, we may try to make progress by modelling the motion of fluid parcels. The simplest model is, for parcel A

$$S_A = \sum_i u_i^* \Delta t \quad \text{when } \langle u_i u_j \rangle = 0 \quad i \neq j \\ \langle u_i^* \rangle = \langle u' \rangle$$


Physical
hence
characteristic
statistically

If we blindly use Taylor's formula

$$K = -\langle u \cdot \sigma \rangle = -\langle u' \rangle \Delta t$$

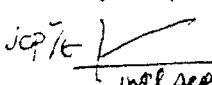
but what does this mean?

If all parcels move independently, like molecules; $\langle u_i^* u_j^* \rangle = 0$
then ^{intuitively} an initial blob of parcel diffuse out with $K = -\langle u' \rangle \Delta t$.
This is really just $K = -u \langle u \Delta t \rangle = -u \times mfp = mfp^2 / \Delta t$

If all parcels move exactly the same way $u^A = u^B \dots$
we still get $K = -\langle u' \rangle \Delta t$ but now an initial blob
never change shape, it just jiggles around. This
isn't what we intuitively mean by diffusion even though
a single blob ~~changes~~ does grow.

The ocean is in between for parcel separated by
mean velocity $u^A = u^B$ — it is not clear that $\langle u_i^* u_j^* \rangle$
ever is zero, altho much depends on what $\langle \rangle$ hides.
To be sure we are really looking at parcels being
taken apart we need to think about
 $\langle (\sigma^A - \sigma^B)^2 \rangle$.

$$\text{In our model } \langle (\sigma^A - \sigma^B)^2 \rangle = \cancel{\text{constant}} \cancel{\text{distance}} \cancel{\text{time}} \cancel{\text{P.D.}} \\ = \langle \sum_i (u_i^* \Delta t - u_j^* \Delta t)^2 \rangle = N \Delta t \langle u'^2 \rangle$$

the separation/ $(\text{time} = \Delta t)$ = $\langle u' \rangle \Delta t = \text{constant}$. But not so
in e.g. Stommel's experiment.  not 