Review of Lynne’s last lecture:

What makes the ocean move?

$$\text{Force} = \text{mass} \times \text{acceleration}$$

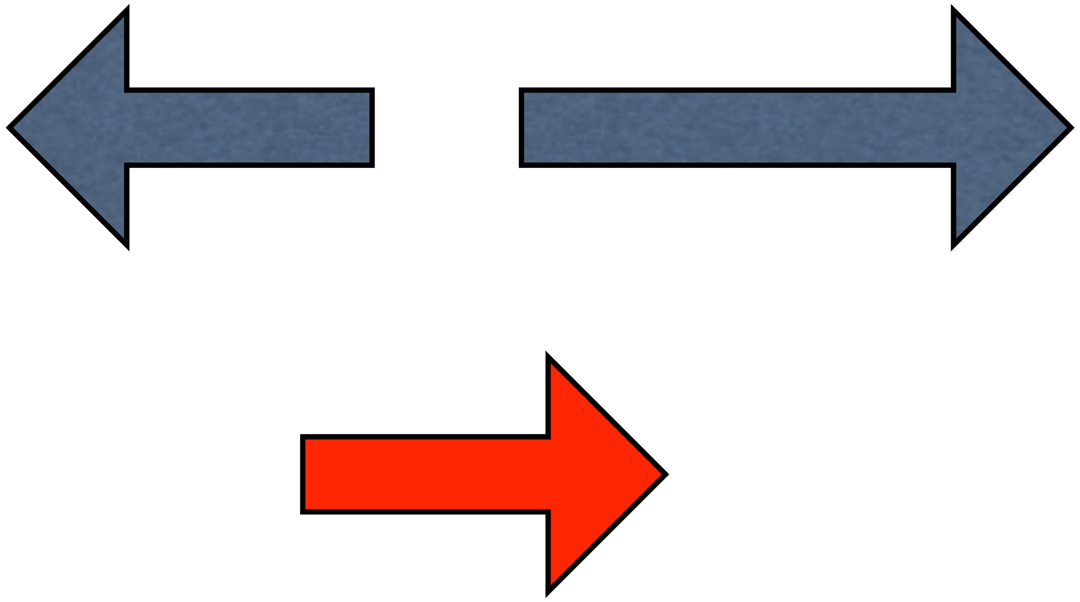
$$F = m \times a$$

$$a = dv / dt = \text{rate of change in velocity}$$

$$\boxed{\text{acceleration} = [\text{Sum of Forces}] / \text{mass}}$$

acceleration = [Sum of Forces] / mass

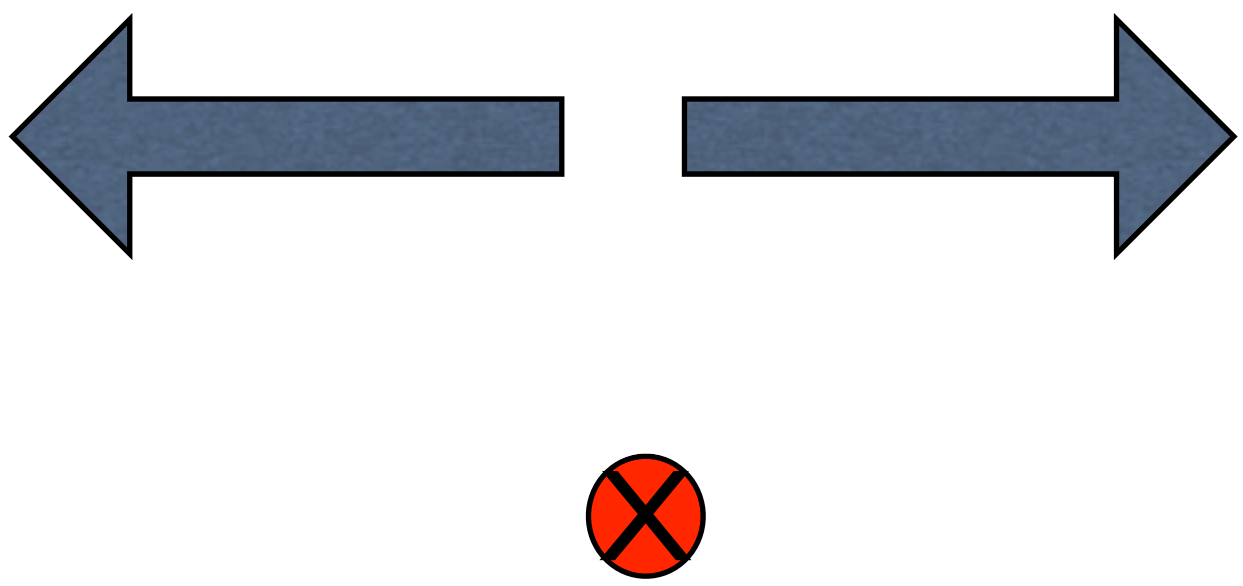
Force 1 + Force 2 = ??



Net force, so accelerate to the right

acceleration = [Sum of Forces] / mass

Force 1 + Force 2 = ??



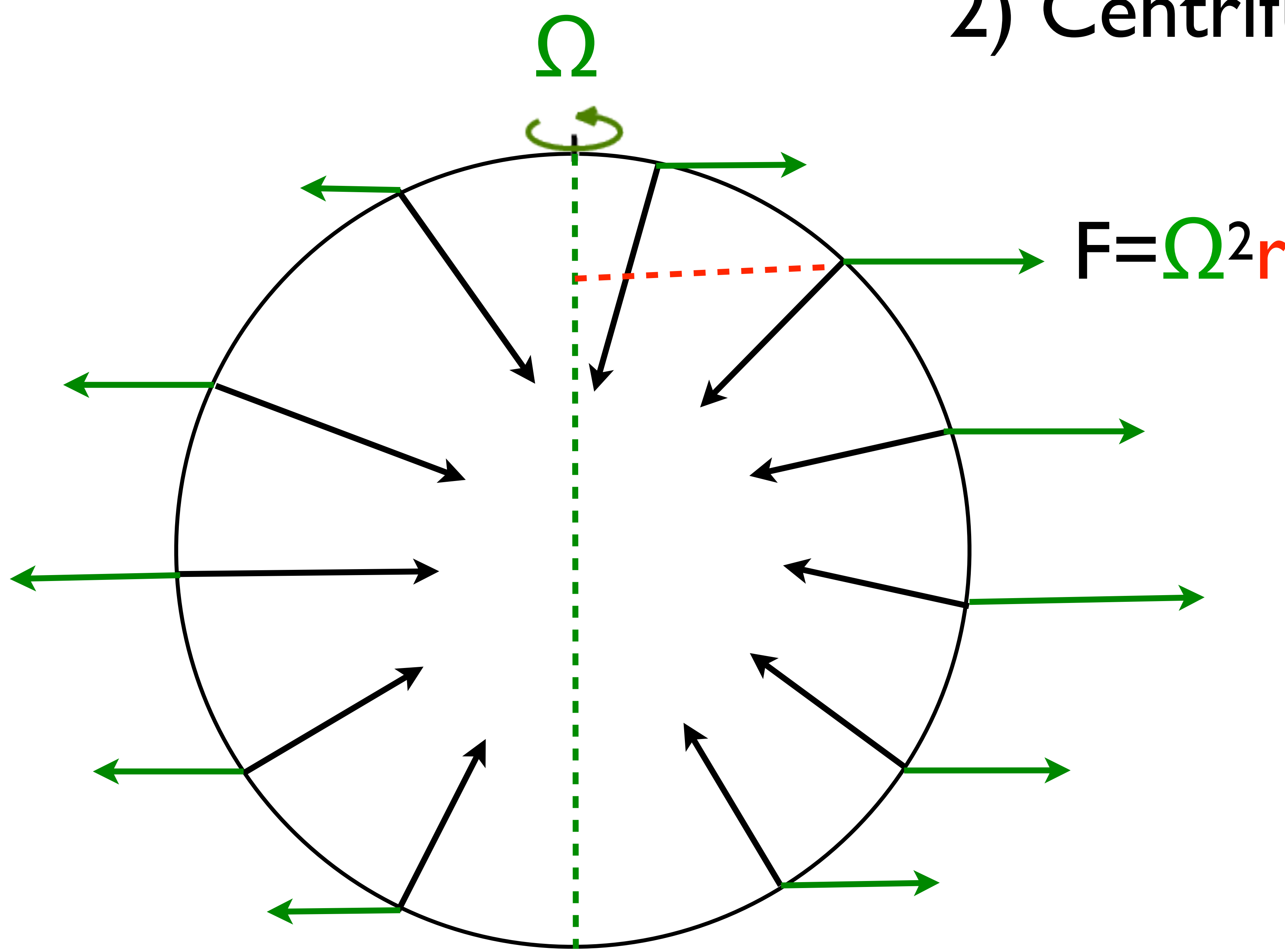
NO net force, so no acceleration

But still could be velocity, it's just that velocity isn't CHANGING

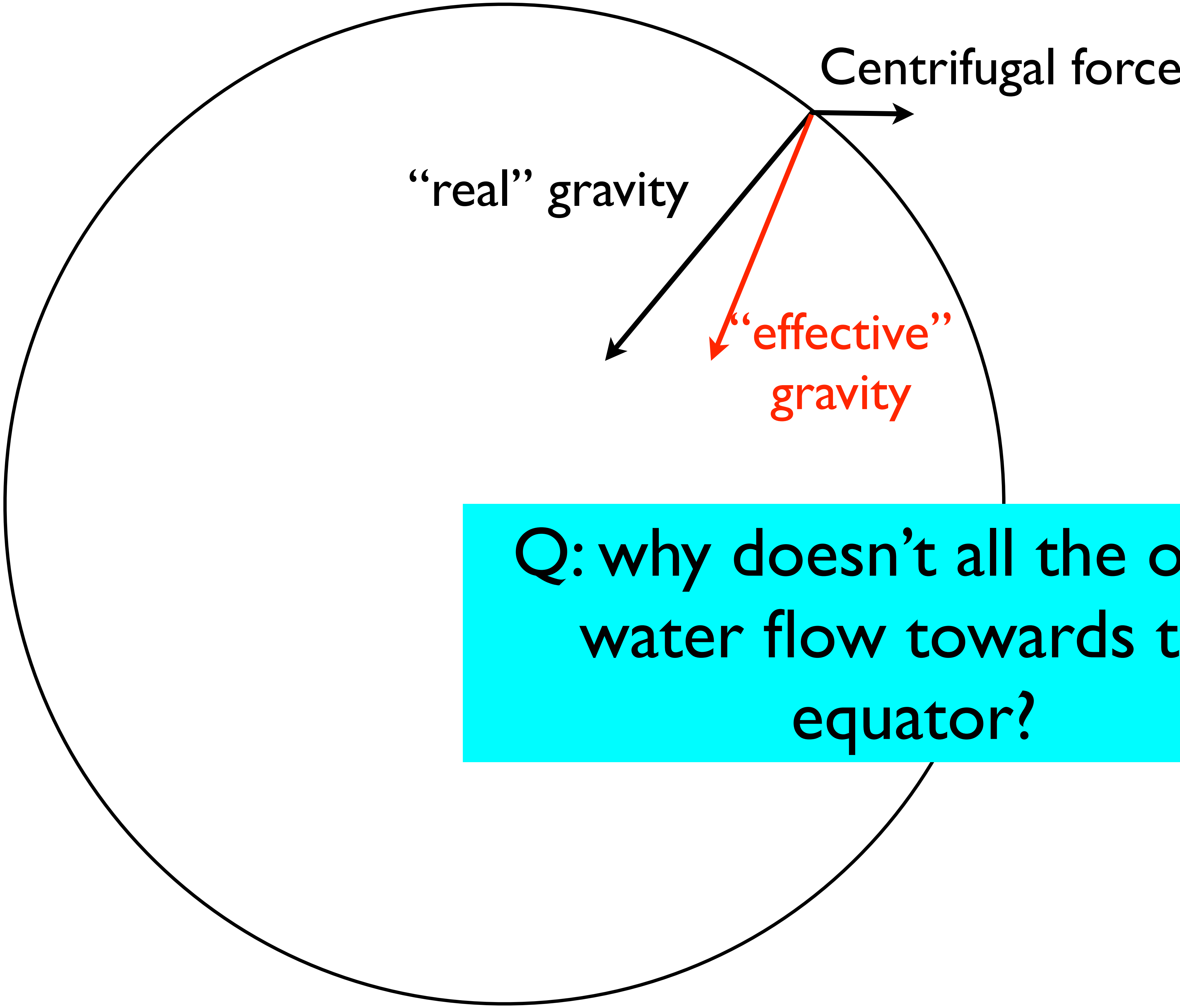
Forces acting on the earth overall

1) Gravity

2) Centrifugal force

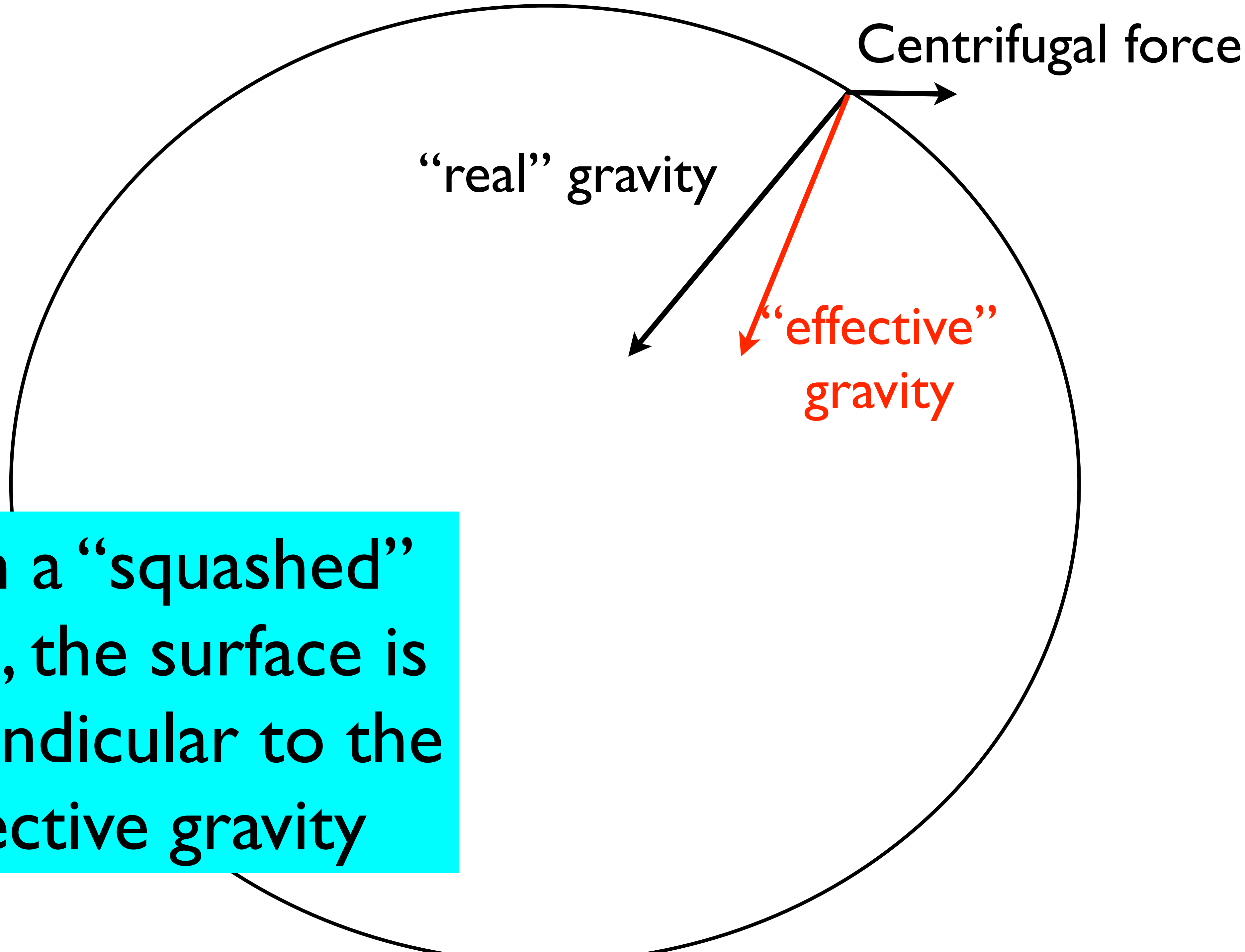


Zooming in



Q: why doesn't all the ocean water flow towards the equator?

Answer: it does! Or rather the whole earth has reshaped itself

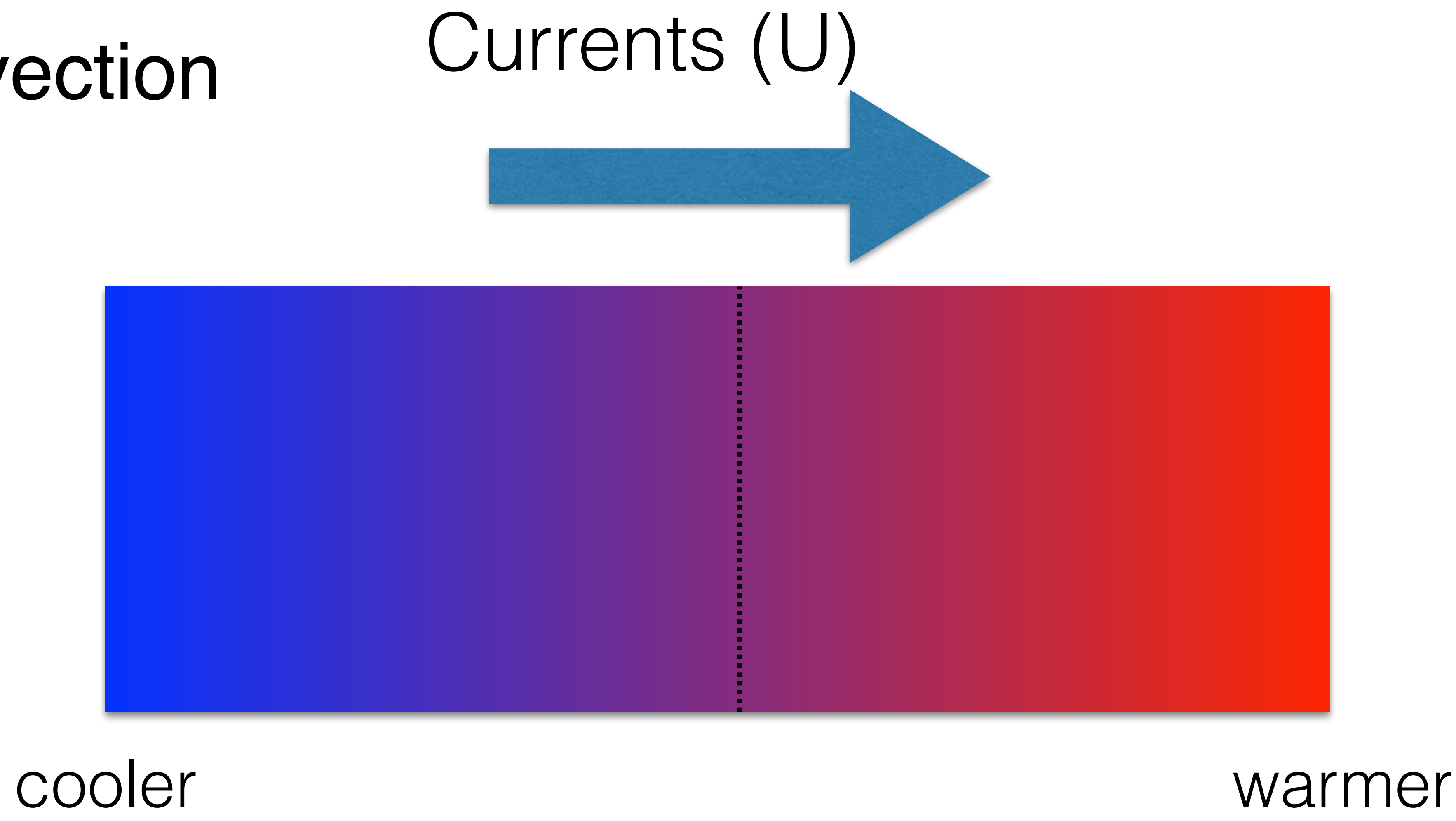


With a "squashed" earth, the surface is perpendicular to the effective gravity

Sideways step - rate of change of temperature

$$\frac{dT}{dt} = \quad ?$$

1) Advection

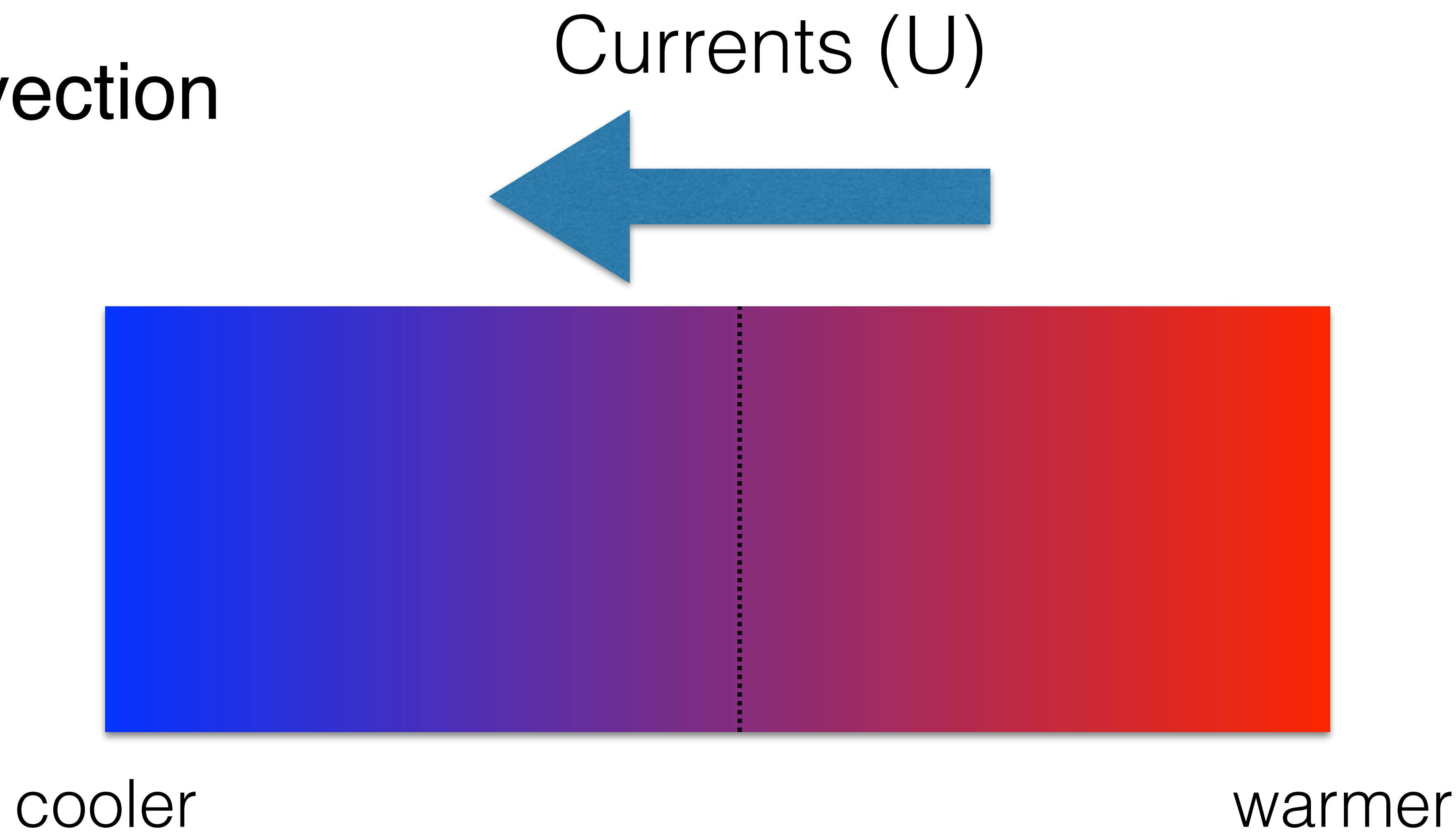


$$\frac{dT}{dx} > 0$$

Sideways step - rate of change of temperature

$$\frac{dT}{dt} = -U\frac{dT}{dx}$$

1) Advection



$$\frac{dT}{dx} > 0$$

Sideways step - rate of change of temperature

$$\frac{dT}{dt} = \quad ?$$

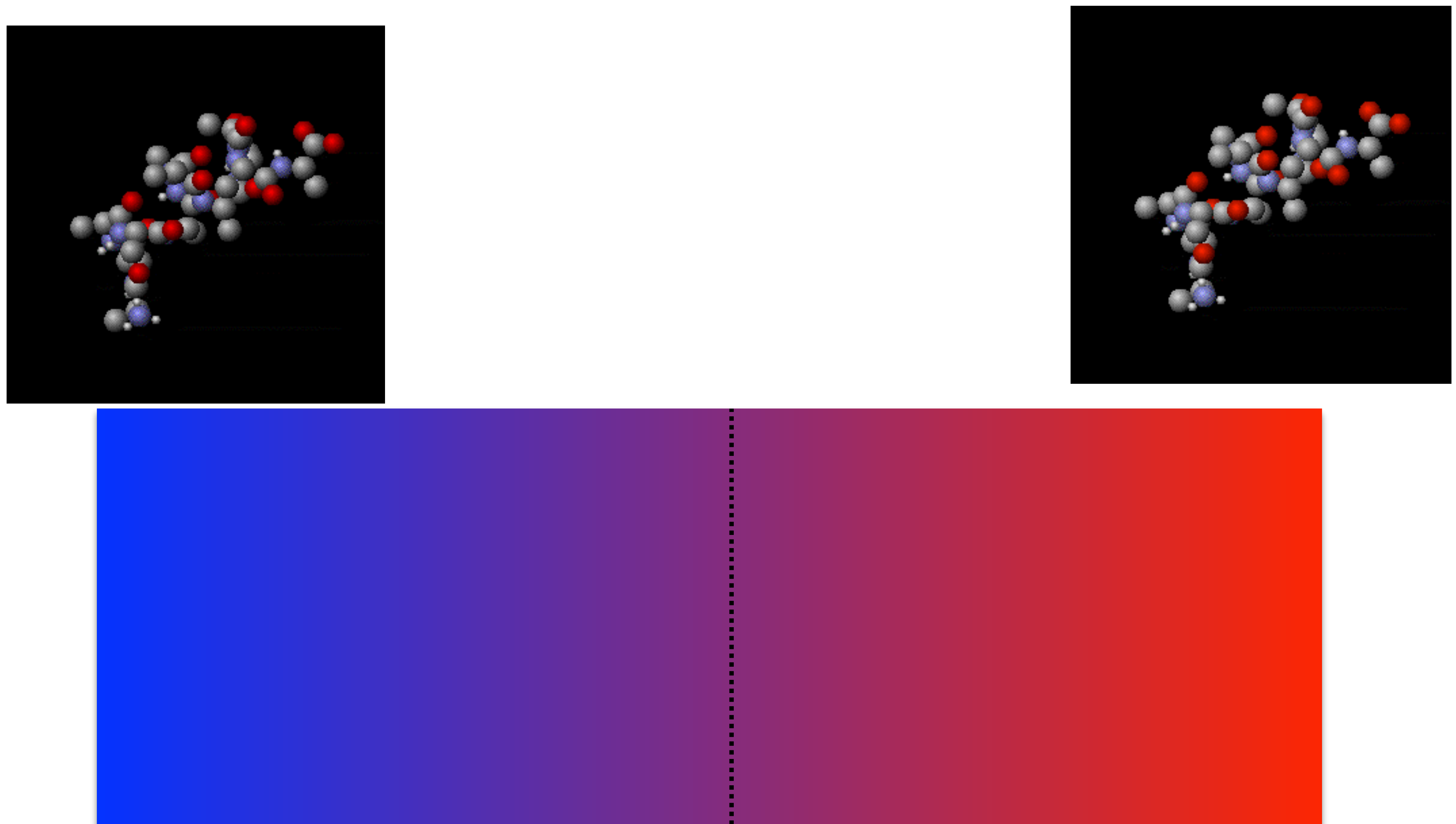
2) Diffusion

$$\frac{dT}{dx} > 0$$

Sideways step - rate of change of temperature

$$\frac{dT}{dt} = \quad ?$$

2) Diffusion



cooler

warmer

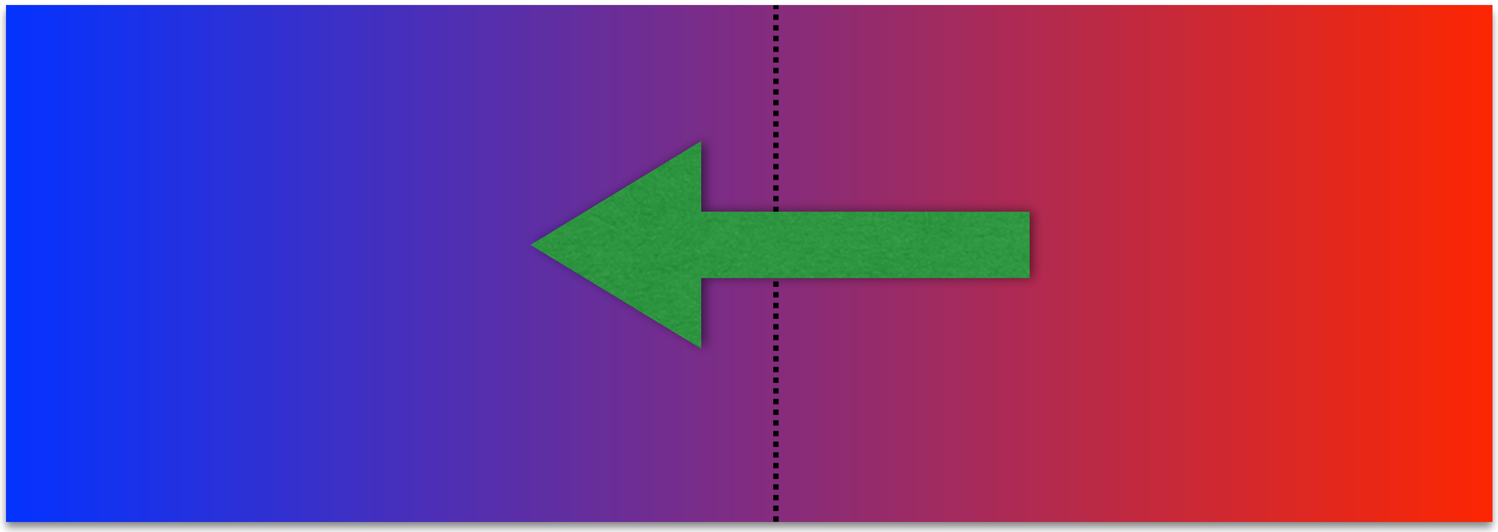
$$\frac{dT}{dx} > 0$$

Sideways step - rate of change of temperature

$$Flux = -\kappa \frac{dT}{dx}$$

2) Diffusion

Flux goes DOWN GRADIENT



cooler

warmer

$$\frac{dT}{dx} > 0$$

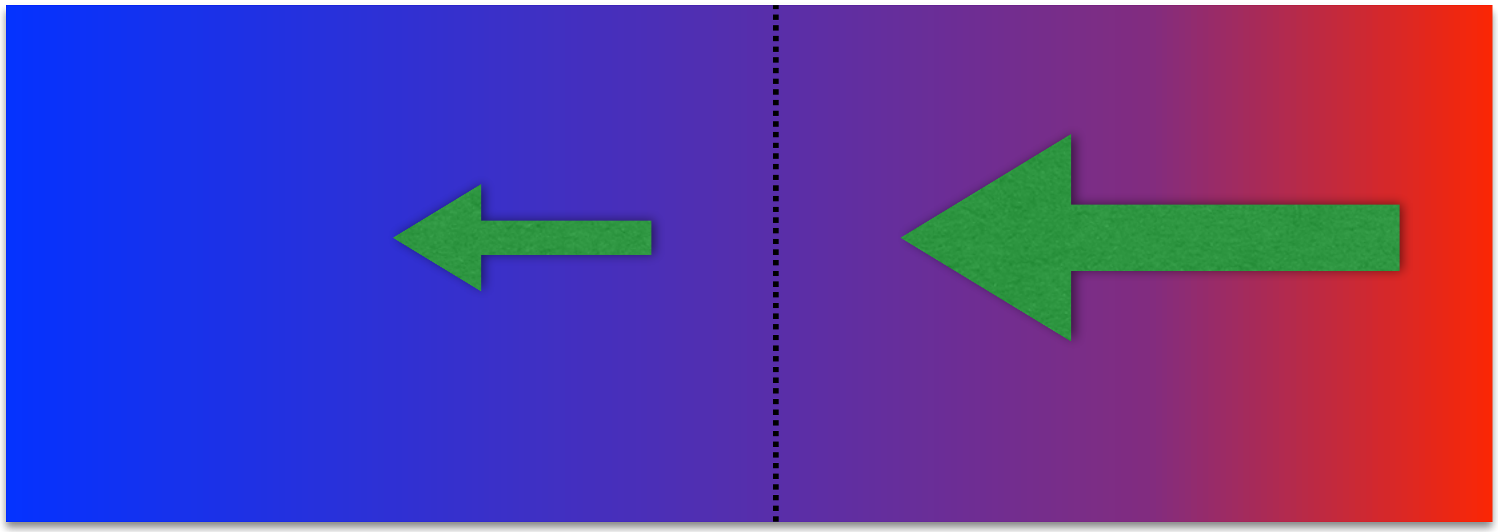
Sideways step - rate of change of temperature

Convergence or divergence of fluxes

$$\frac{dT}{dt} = -\frac{d(\mathrm{Flux})}{dx}$$

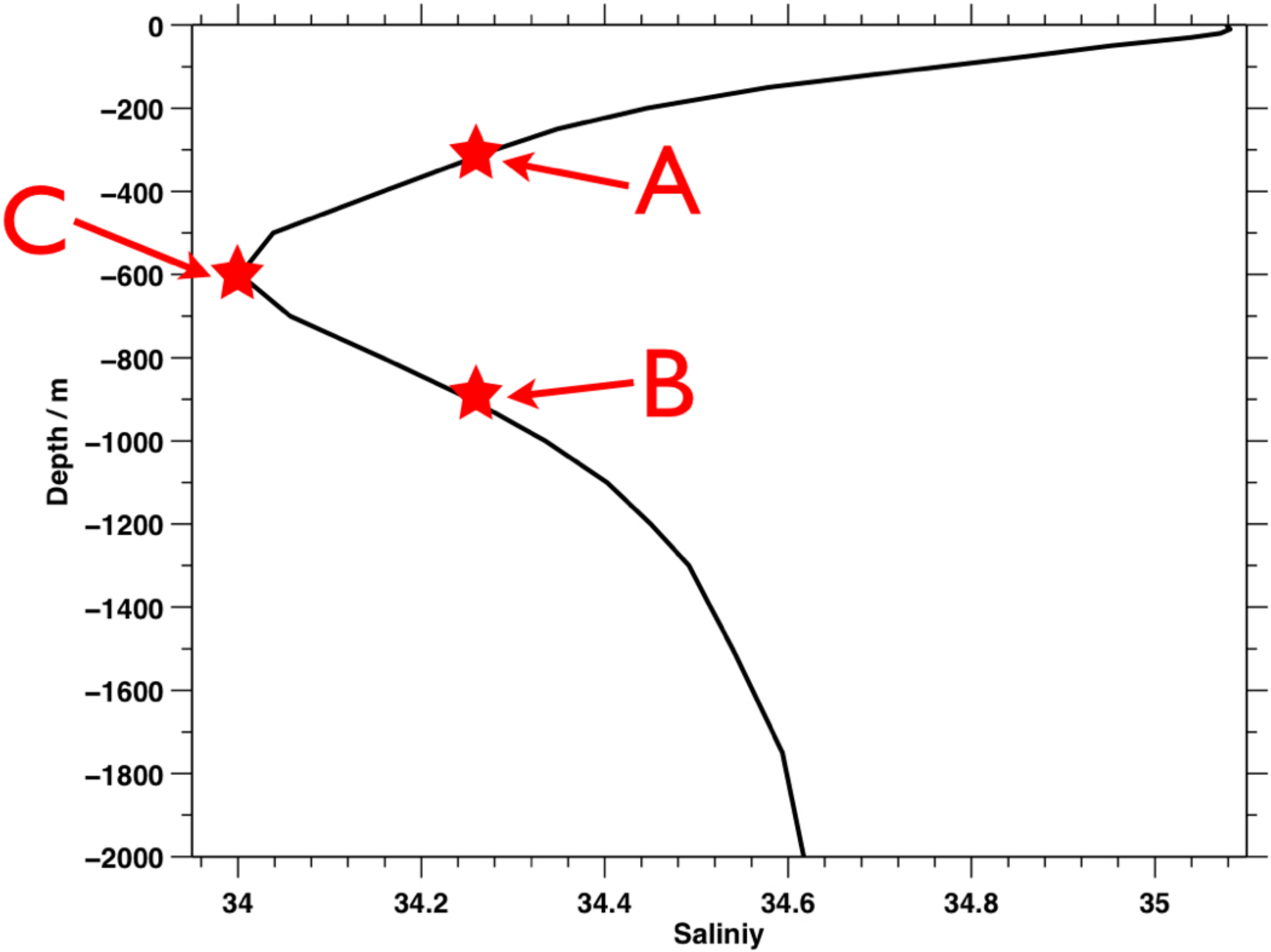
$$Flux = -\kappa \frac{dT}{dx}$$

$$\frac{dT}{dt} = \kappa \frac{d^2T}{dx^2}$$



cooler warmer

$$\frac{dT}{dx} > 0$$


0
−200
−400
−600
−800
−1000
−1200
−1400
−1600
−1800
−2000
Depth / m
34
34.2
34.4
34.6
34.8
35
Saliniy
A
B
C

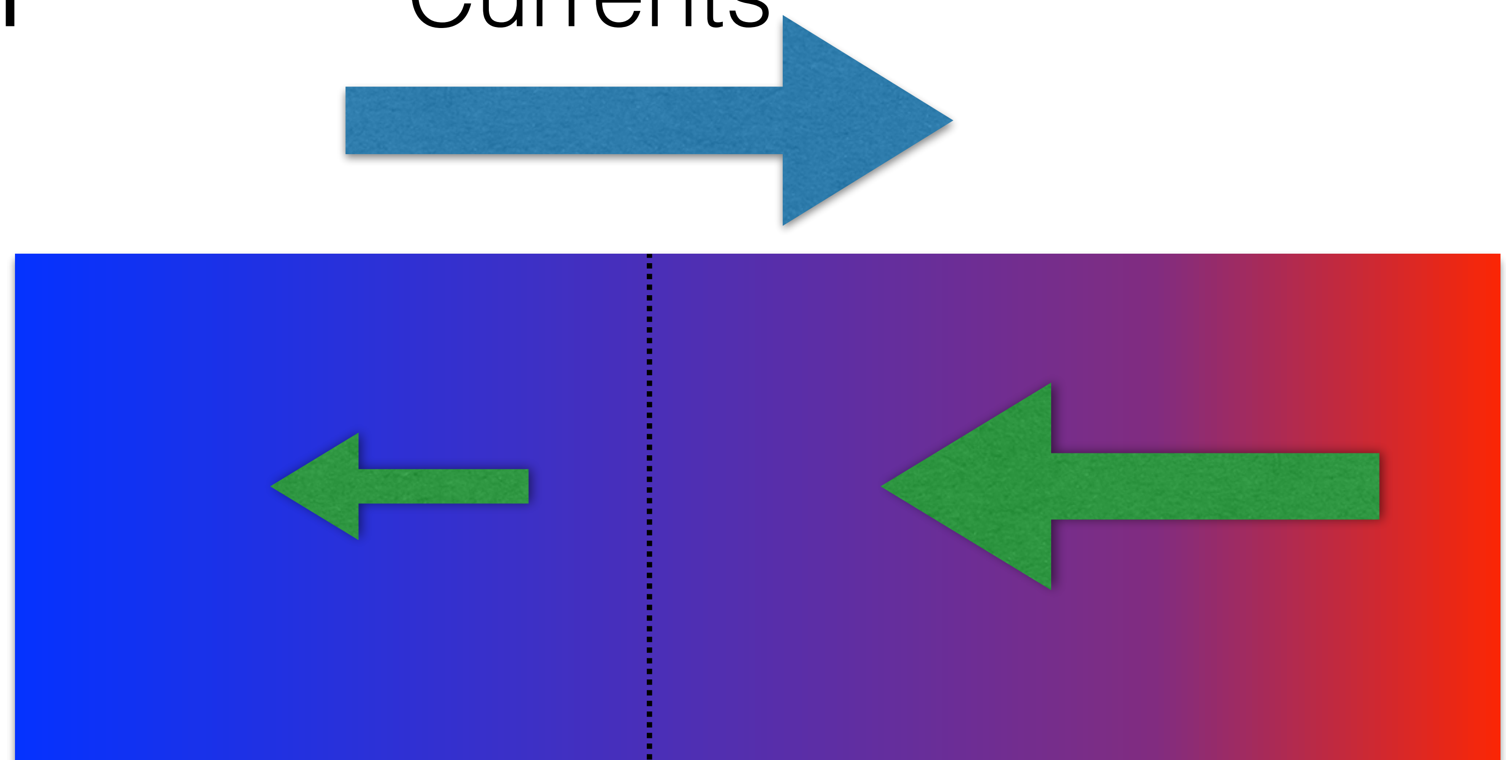
Sideways step - rate of change of temperature

$$\frac{dT}{dt} = -u\frac{dT}{dx} + \kappa\frac{d^2T}{dx^2}$$

1) Advection

Currents

2) Diffusion



cooler

warmer

$$\frac{dT}{dx} > 0$$

Sideways step - rate of change of temperature

Expanding to all directions

$$\frac{dT}{dt} = -u\frac{dT}{dx} - v\frac{dT}{dy} - w\frac{dT}{dz} + \kappa(\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} + \frac{d^2T}{dz^2})$$

$$\frac{dT}{dt} = -\vec{u} \cdot \nabla T + \kappa \nabla^2 T$$

$$\frac{dT}{dt} + \vec{u} \cdot \nabla T = \kappa \nabla^2 T$$

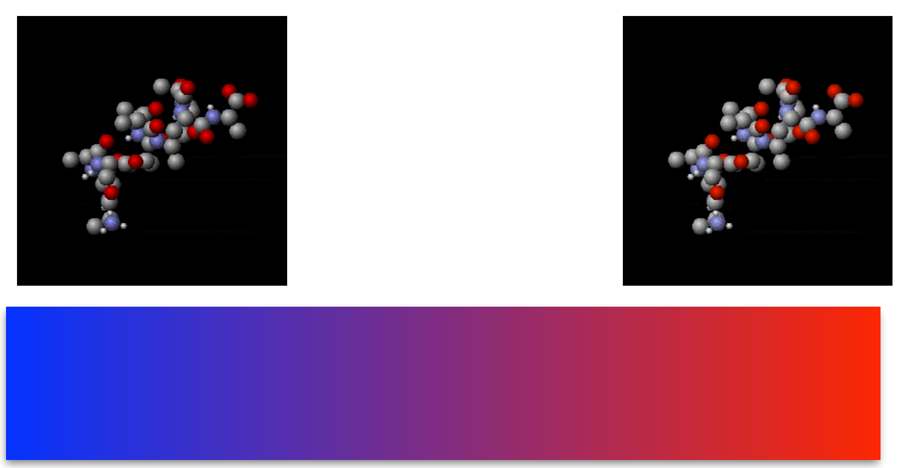
$$\frac{DT}{Dt} = \kappa \nabla^2 T$$ +Source/ Sink

Following a parcel of fluid, the only things that can change it are diffusion. True?

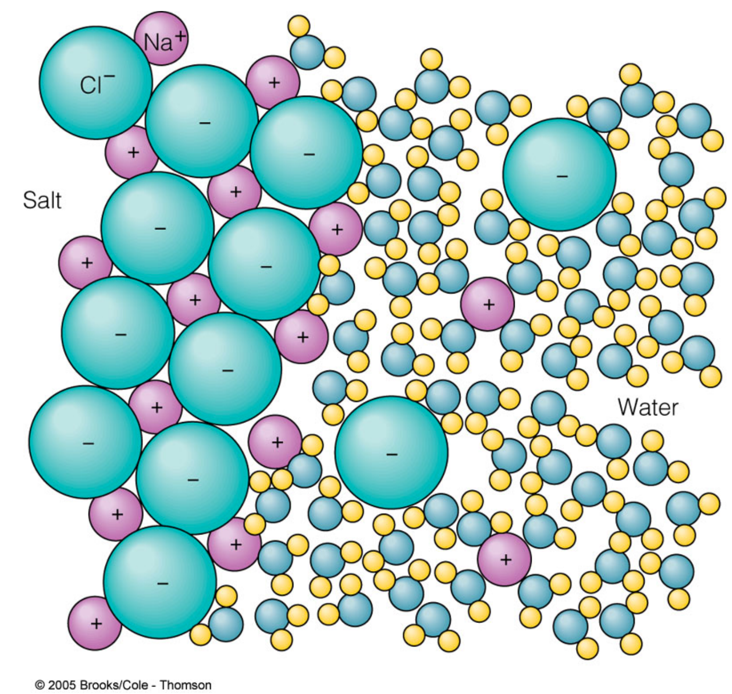
What about salinity?

$$\frac{DT}{Dt} = \kappa \nabla^2 T + \text{Source/Sink}$$

Should the equation be the same for salt?



cooler warmer



saltier fresher

What about salinity?

$$\frac{DT}{Dt} = \kappa_T \nabla^2 T \text{ +Source/Sink}$$

Should the equation be the same for salt? No! Salt diffuses MUCH more slowly.

$$\frac{DS}{Dt} = \kappa_S \nabla^2 S \text{ +Source/Sink}$$

$\kappa_T \sim 10^{-7}$ m^2/s

$\kappa_S \sim 10^{-9}$ m^2/s

(book, chapter S7)

Net lessons for temperature/salinity equations

$$\frac{DT}{Dt} = \kappa_T \nabla^2 T$$ +Source/Sink

$$\frac{DS}{Dt} = \kappa_S \nabla^2 S$$ +Source/Sink

- At any one place (d/dt), the change in temperature is due to 1) advection (movement) of water when there is a temperature gradient, 2) diffusion to/from your cooler/warmer neighbors, and 3) sources/sinks (solar heating, etc)

- If you are following along with a water parcel (D/DT) the advection term is gone, and only diffusion and sources/sinks change your temperature.

Now moving on to “momentum equations”

Force = mass × acceleration

F = m × a

a = dv / dt = rate of change in velocity

In the ocean we don’t consider total mass (it’s not a discrete thing), but density = mass per unit volume

$$\rho \frac{dU}{dt} = \text{Sum of forces acting on that bit of volume, or parcel of water}$$

$$\frac{dU}{dt} = \frac{1}{\rho}(\text{Sum of forces acting on that bit of volume, or parcel of water })$$

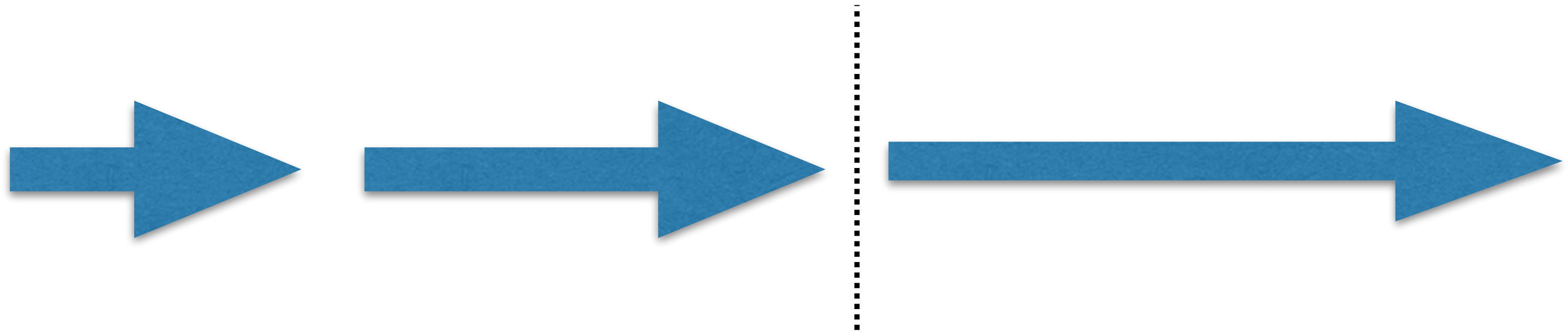
Momentum equation: advective terms

$$\frac{dT}{dt} = -u\frac{dT}{dx} + \kappa\frac{d^2T}{dx^2}$$

The first couple terms look similar to those for temperature

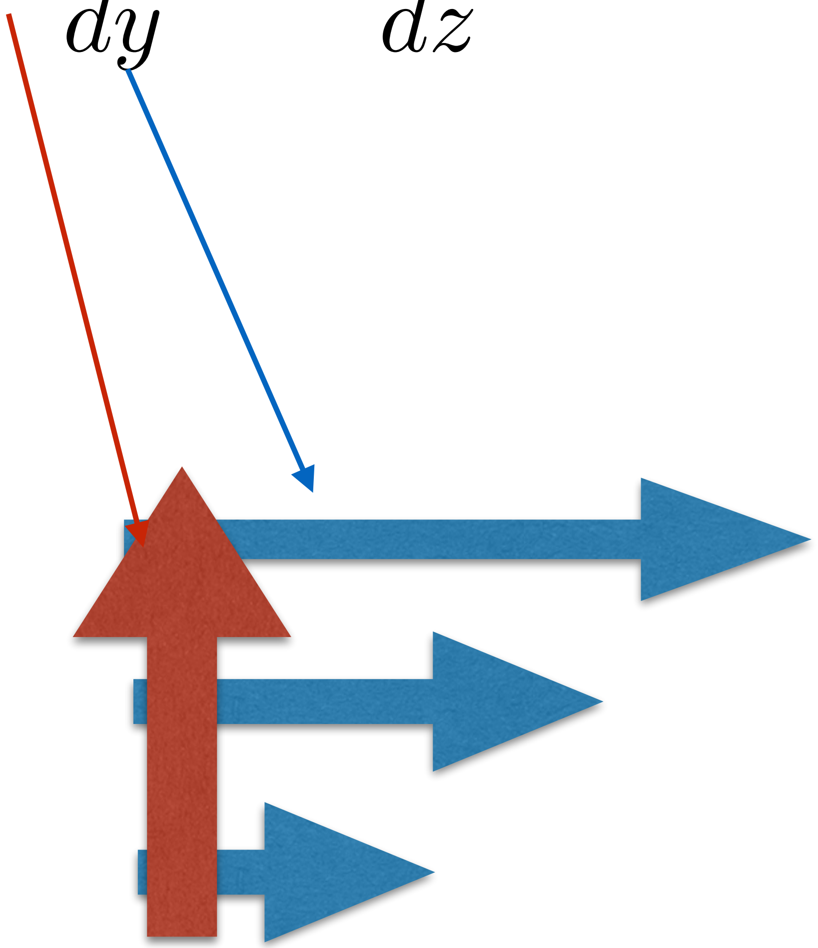
$$\frac{du}{dt} = -u\frac{du}{dx} \quad + \ldots..$$

What does advection mean here?



Momentum equation: advective terms

$$\frac{du}{dt} = -u\frac{du}{dx} - v\frac{du}{dy} - w\frac{du}{dz} \quad +\ldots..$$

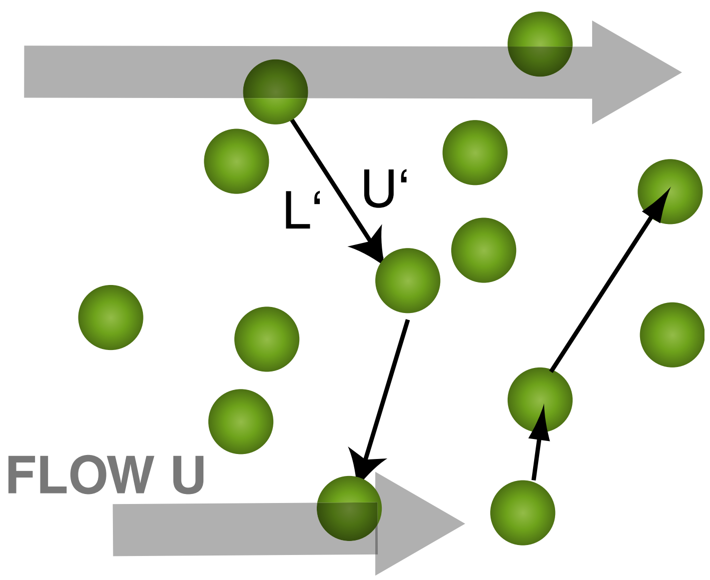


$$\frac{Du}{Dt} = +\ldots..$$

Momentum equation: viscous terms

$$\frac{DT}{Dt} = \kappa_T \nabla^2 T \quad \text{+Source/Sink}$$

What’s the equivalent term here? How do you “diffuse” velocity or momentum?



$$\frac{Du}{Dt} = \nu \nabla^2 u \;\; + \ldots\ldots$$

ν is the <u>kinematic viscosity</u>

$\sim 1.8 \; 10^{-6} \; m^2/s$

Momentum equation: Stress / drag

At the surface or bottom of the ocean, instead of viscous stress (the ocean dragging on itself), you have

Bottom drag: friction with the sea-floor slows down the flow. The strength of this effect increases with the current speed.

$$\frac{Du}{Dt} = \nu \nabla^2 u + \ldots..$$

$$\frac{Du}{Dt} = -C_D u^2 + \ldots..$$

at the ocean bottom!

Wind Stress: the wind 'drags' the ocean along with it. This speeds up the ocean currents, in the direction of the wind.

$$\frac{Du}{Dt} = \nu \nabla^2 u + \ldots..$$

$$\frac{Du}{Dt} = \frac{1}{\rho}\tau + \ldots..$$

at the ocean surface!

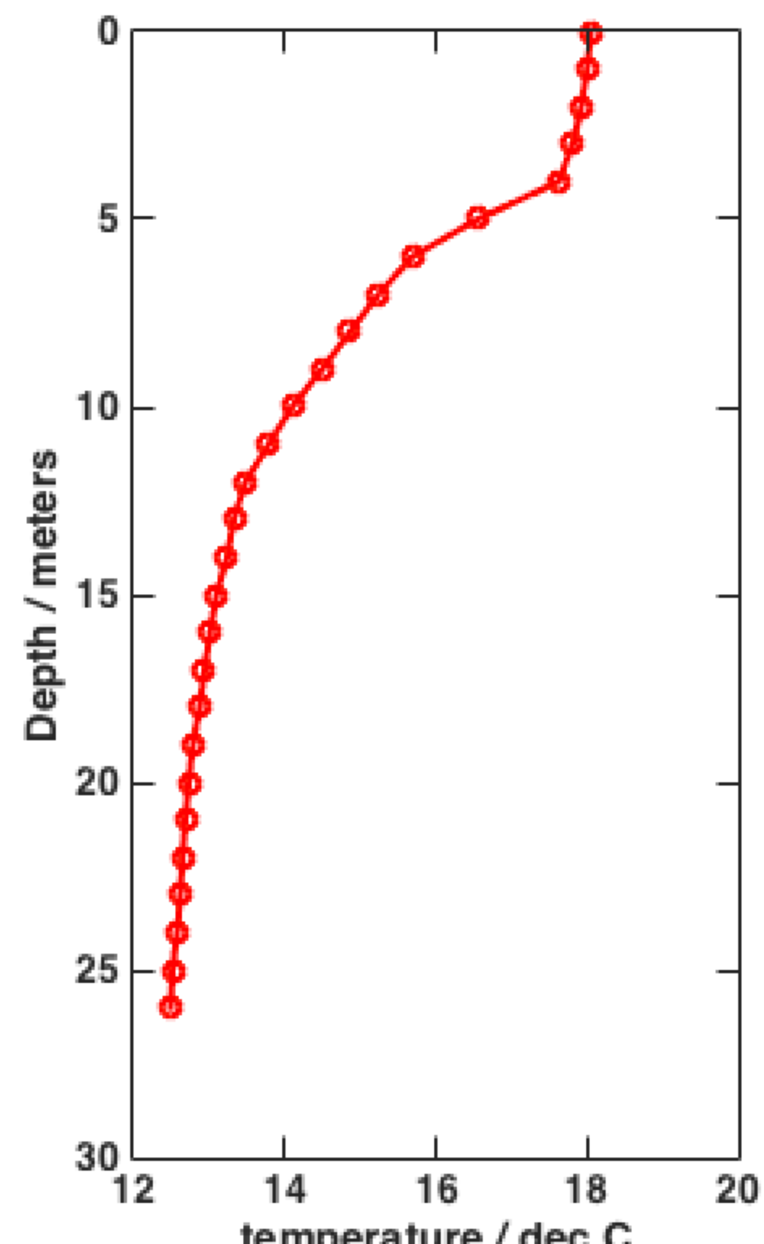
think about some numbers

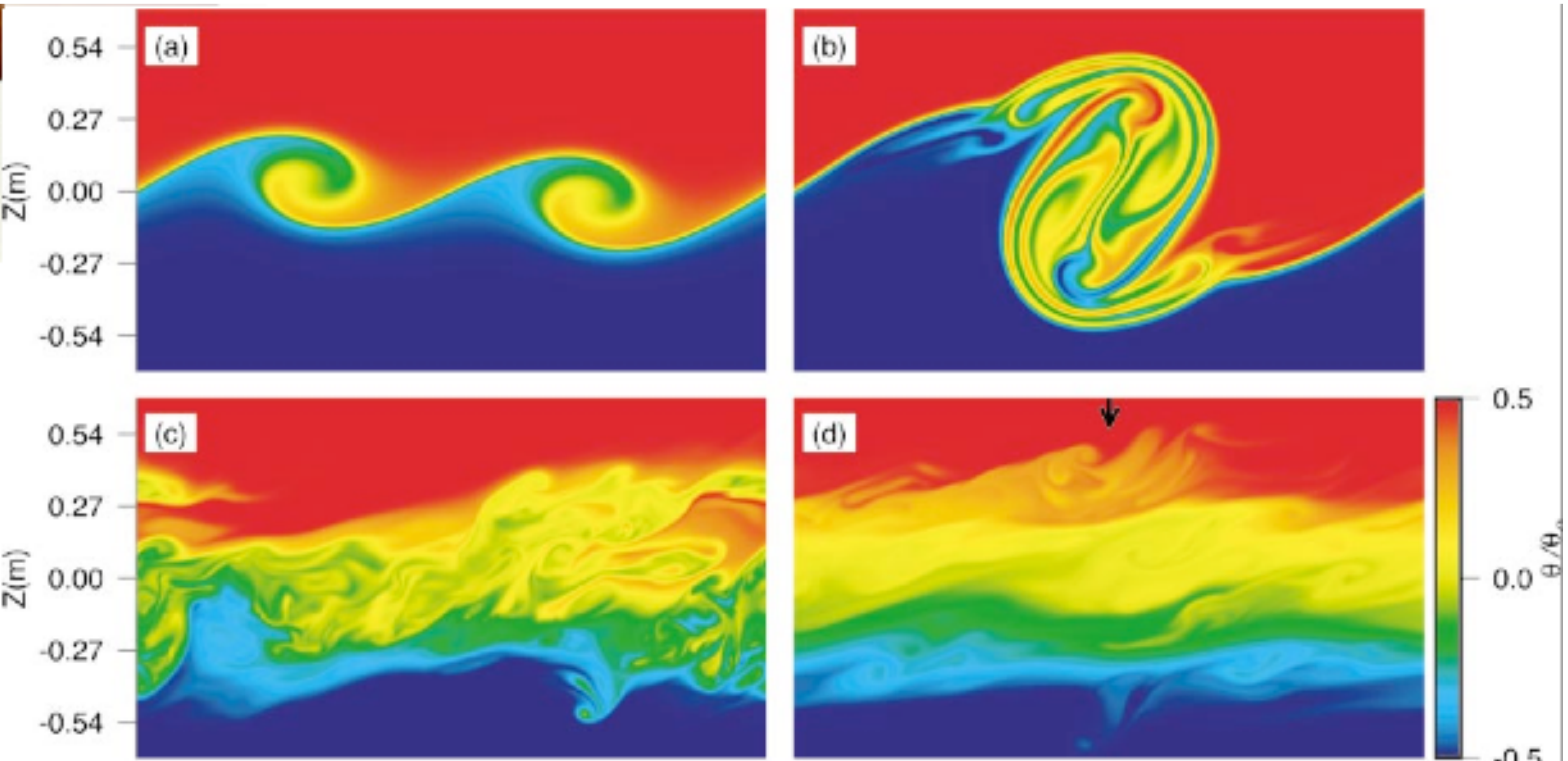
$$\frac{DT}{Dt} = \kappa_T \nabla^2 T + \text{Source/Sink}$$

A good way to get a basic feel for the answer is to look at the rough size and units of different terms

$$\frac{[\text{Temperature}]}{[\text{Time}]} = \kappa_T \frac{[\text{Temperature}]}{[\text{Length}]^2}$$

Think about how long it takes temperature in the profiles on the right to diffuse about 10 meters downwards from the ocean surface , by re-arranging the rough equation above, and using $\kappa_T \sim 10^{-7}$ m²/s



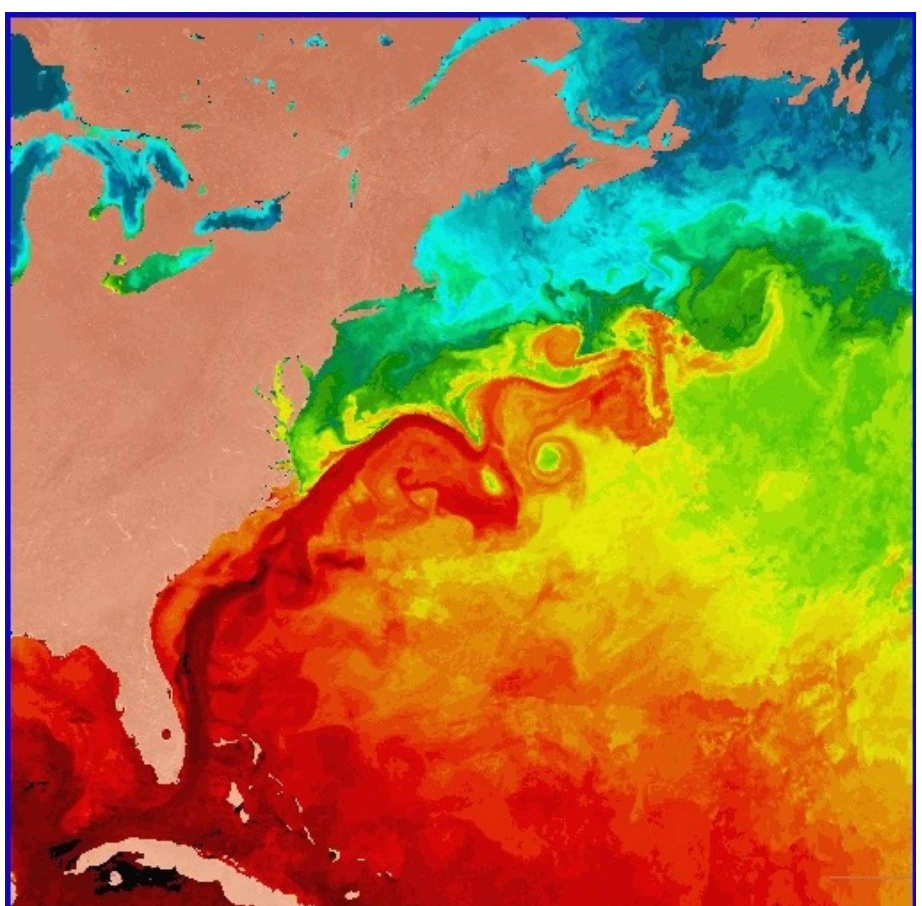


(Bill Smyth)

Stirring and mixing

Vertical stirring and ultimately mixing:

Internal waves on an interface stir fluid, break and mix



Horizontal stirring and ultimately mixing:

Gulf Stream (top): meanders and makes rings (closed eddies) that transport properties to a new location

Eddy viscosity and diffusivity

- Molecular diffusivity and viscosity

  κ_T = 0.0014 cm^2/sec (temperature)

  κ_S = 0.000013 cm^2/sec (salinity)

  ν = 0.018 cm^2/sec at 0°C (0.010 at 20°C)

- Eddy diffusivity and viscosity values for heat, salt, properties are the same size (same eddies carry momentum as carry heat and salt, etc)

  But eddy diffusivities and viscosities differ in the horizontal and vertical

- Eddy diffusivity and viscosity

  A_H = 10^4 to 10^8 cm^2/sec (horizontal) = 1 to 10^4 m^2/sec

  A_V = 0.01 to 10 cm^2/sec (vertical) = 10^{-6} to 10^{-3} m^2/sec

Momentum equation: total

$$\frac{Du}{Dt} = -\frac{1}{\rho}\frac{dp}{dx} + A_H\left(\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2}\right) + A_v\frac{d^2u}{dz^2}$$

$$\frac{Dv}{Dt} = -\frac{1}{\rho}\frac{dp}{dy} + A_H\left(\frac{d^2v}{dx^2} + \frac{d^2v}{dy^2}\right) + A_v\frac{d^2v}{dz^2}$$

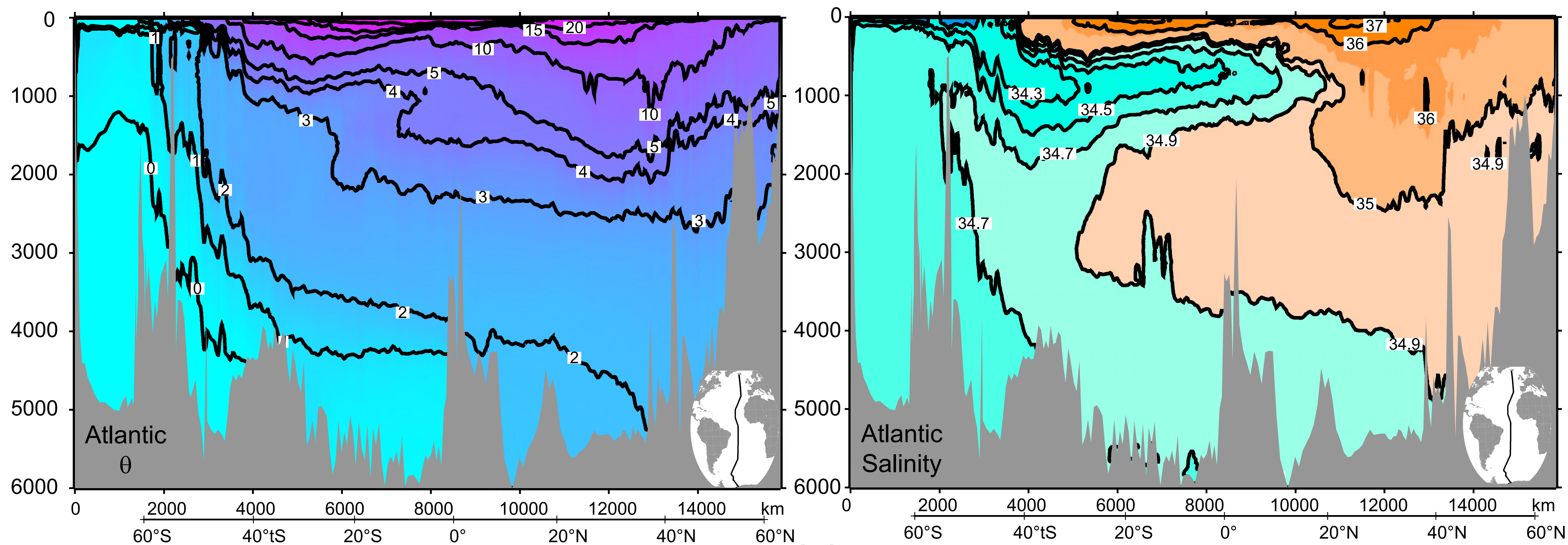
$$\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{dp}{dz} + A_H\left(\frac{d^2w}{dx^2} + \frac{d^2w}{dy^2}\right) + A_v\frac{d^2w}{dz^2} + g$$

Acceleration following a fluid parcel

Pressure differences push water

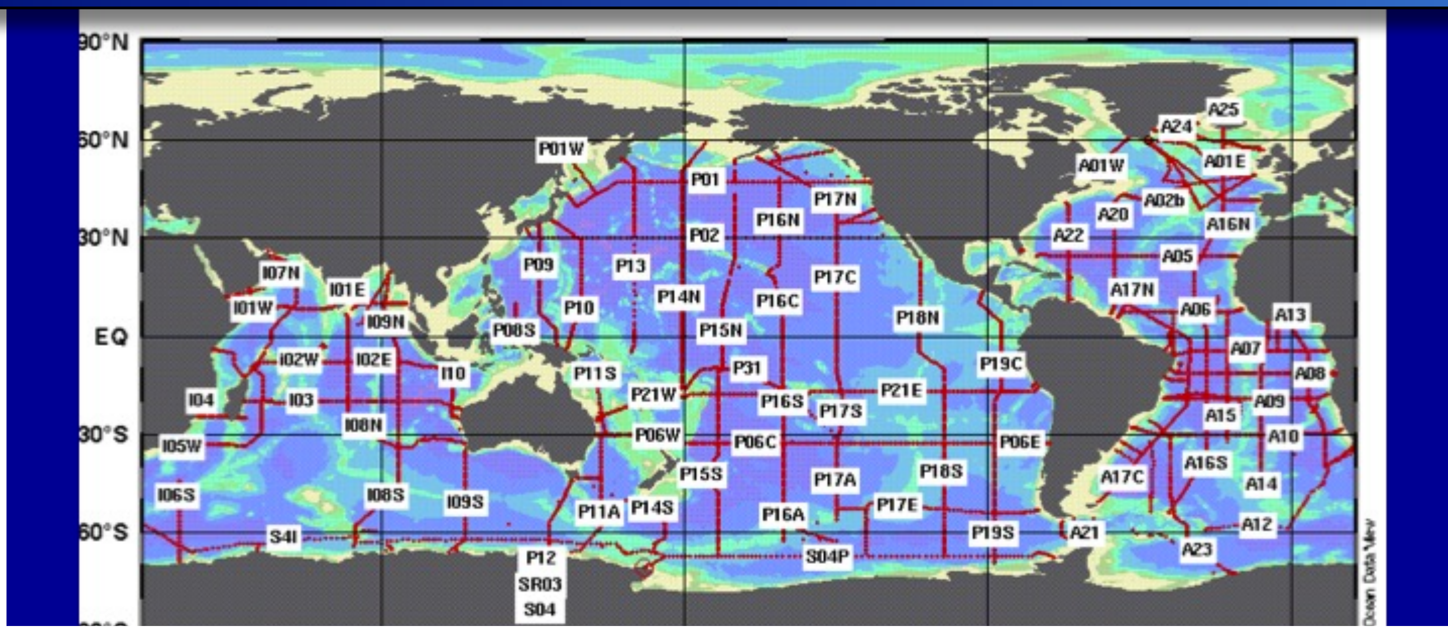
Redistribution of momentum within the ocean, tends to act gradually, sometimes ignored for some problems. But the surface/bottom stress versions are not!!

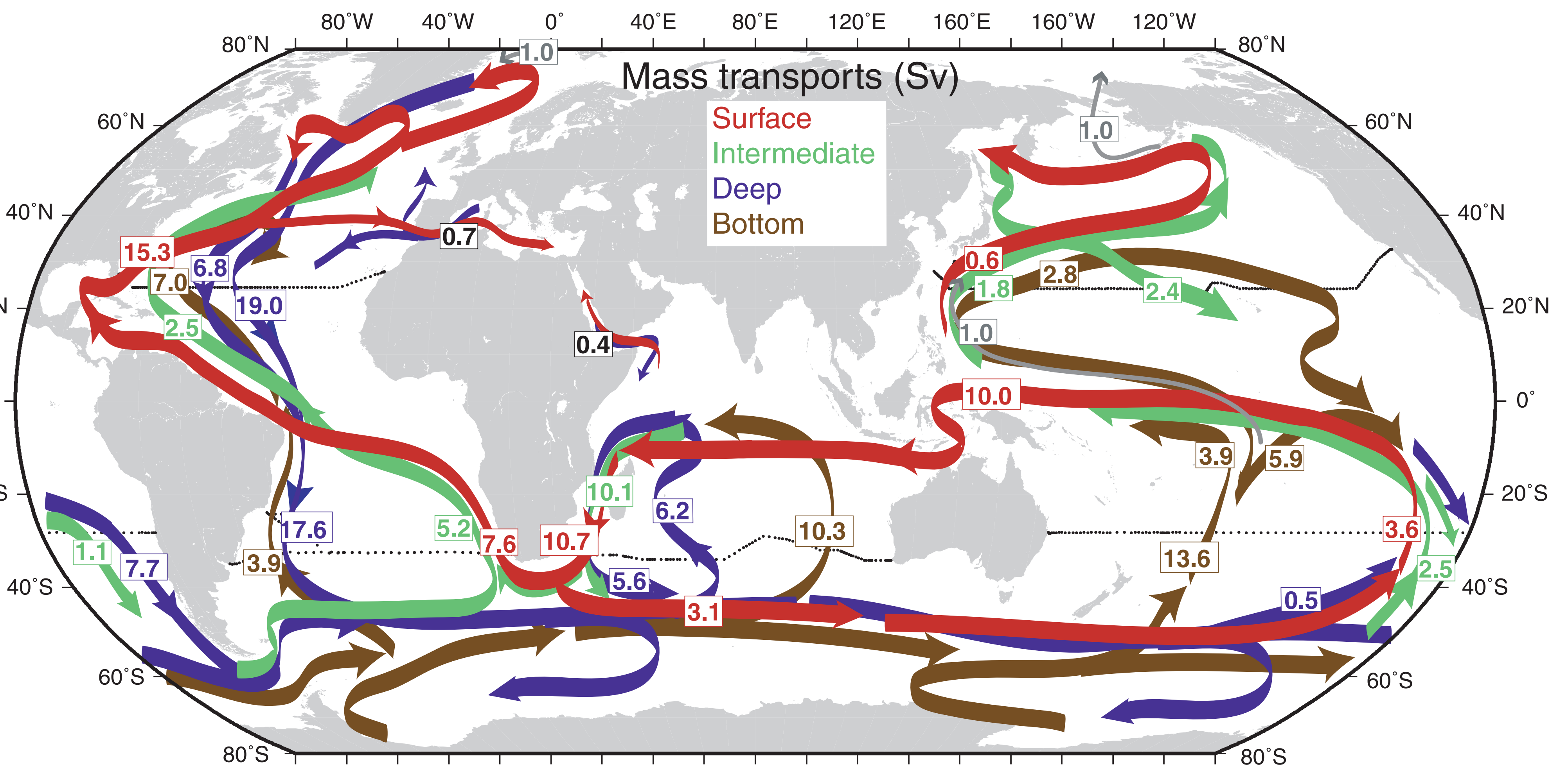
More on the mixing term: let's look at the large-scale, smooth ocean circulation



(Talley et al, 2011)

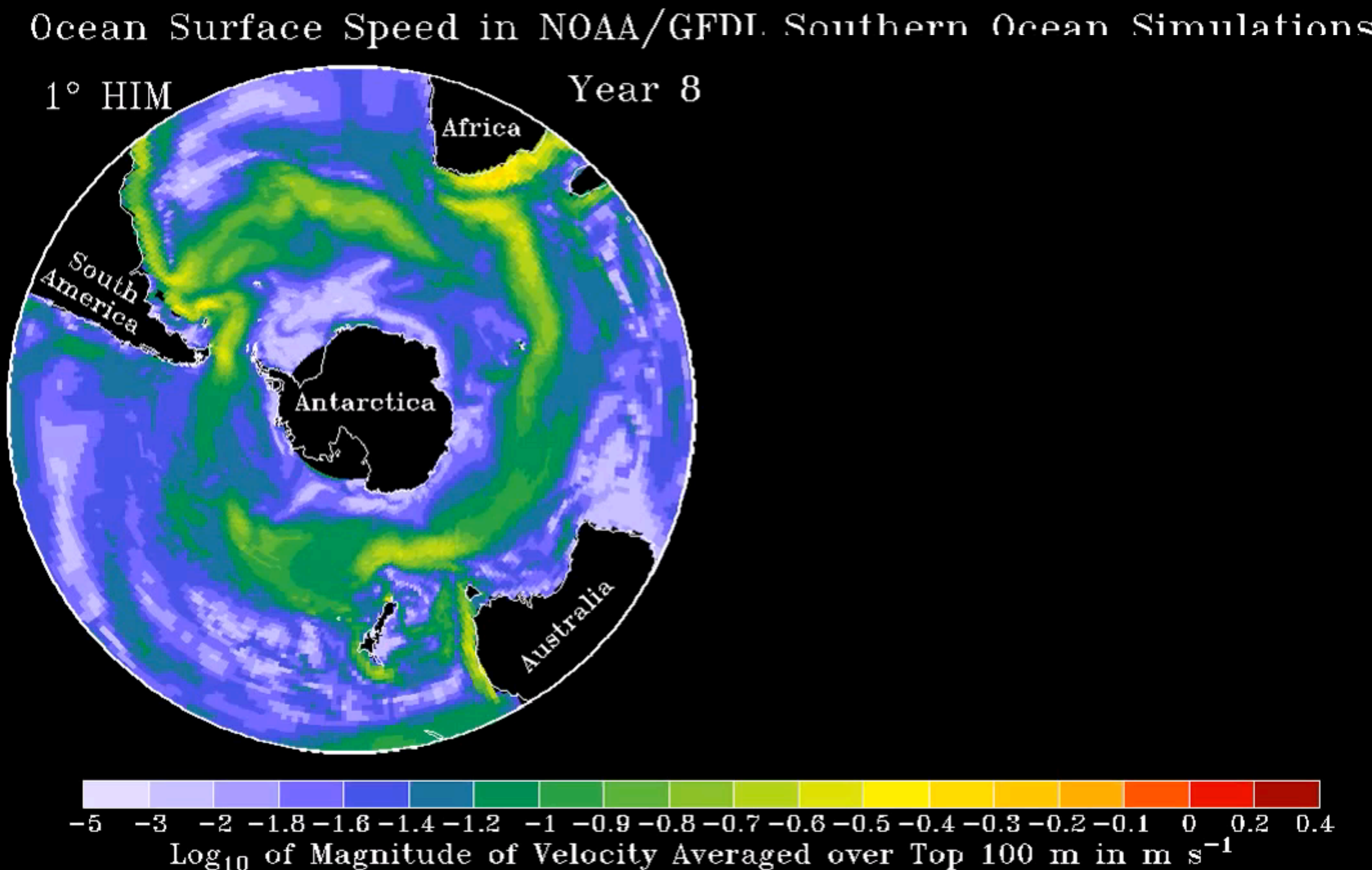
World Ocean Circulation Experiment reveals more complex, global “conveyor belt” patterns

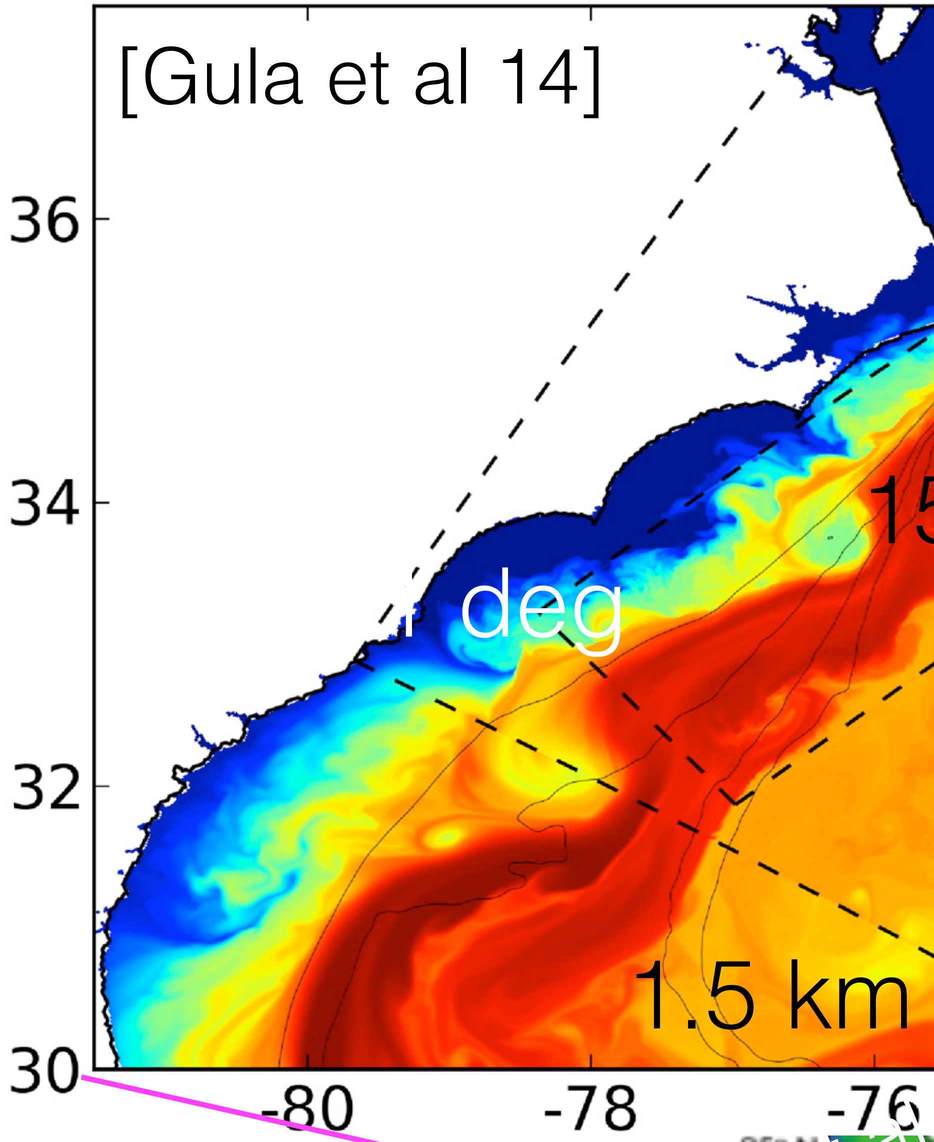


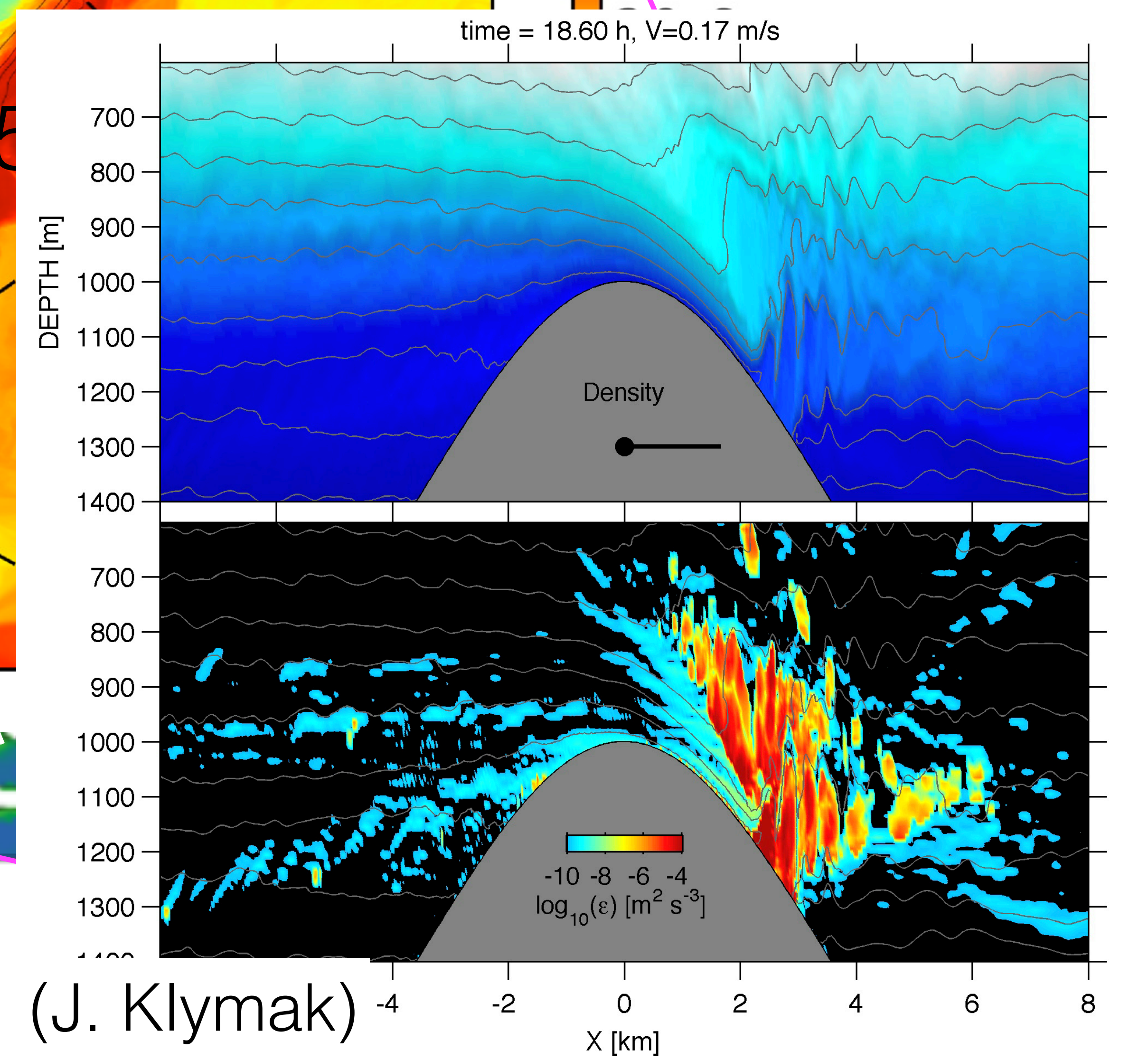


(Talley et al, 2011)

[Hallberg & Gnanadesikan, JPO 2006]







(J. Klymak)

[Saba et al 2016]

With increasing resolution comes increasing complexity

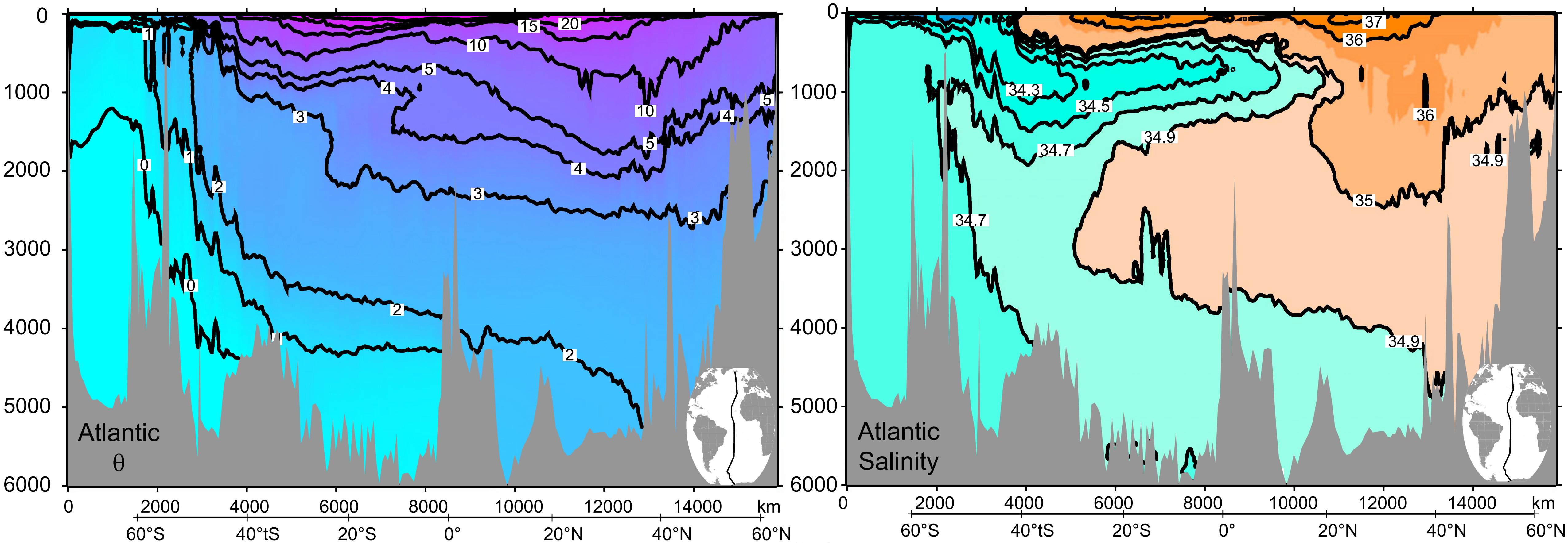
Why care about diapycnal turbulent mixing?

1. Deep and abyssal mixing set deep circulation patterns and energetically drive the MOC.

2. Mixing in the upper ocean controls SST and hence strongly affects air-sea fluxes, in both directions.

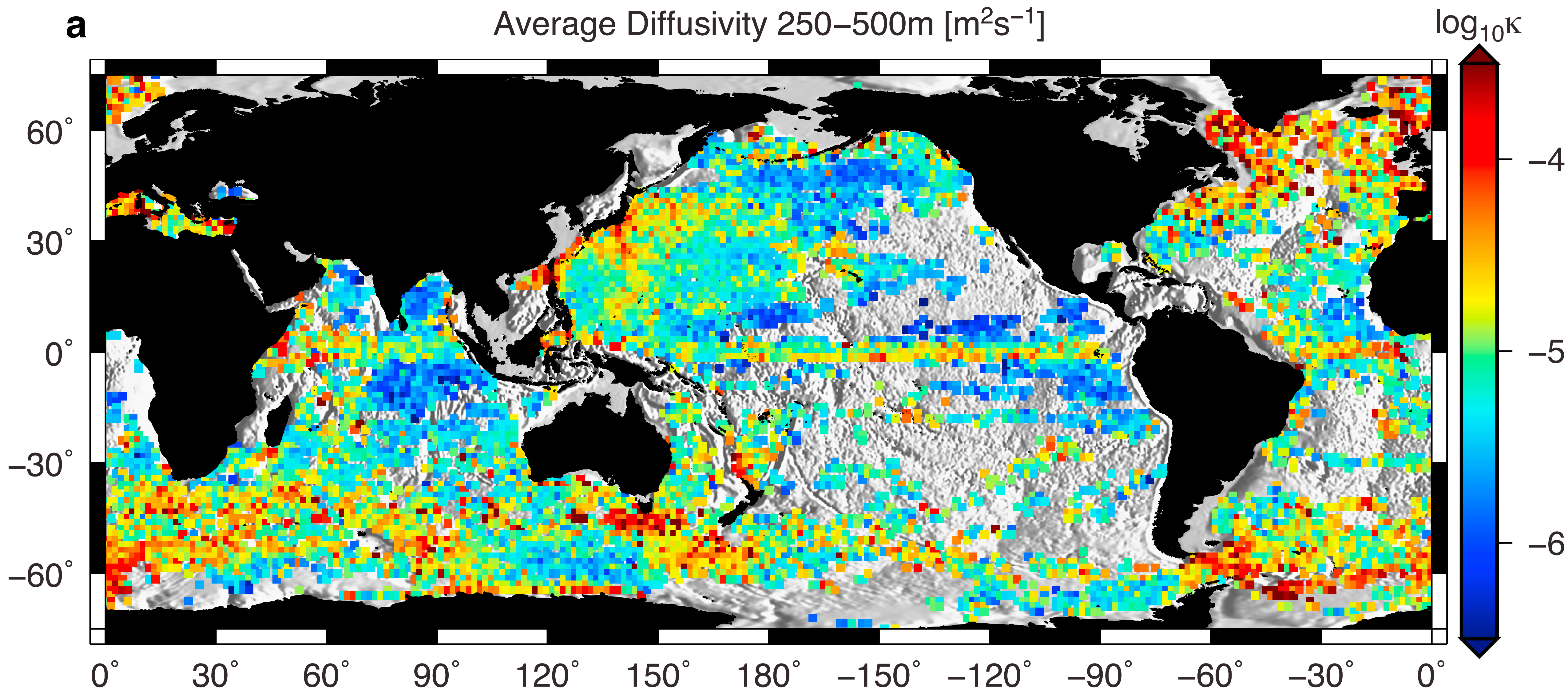
3. Mixing at all depths influences the distributions of tracers, dissolved gasses, nutrients.

Large-scale, smooth ocean circulation



(Talley et al, 2011)

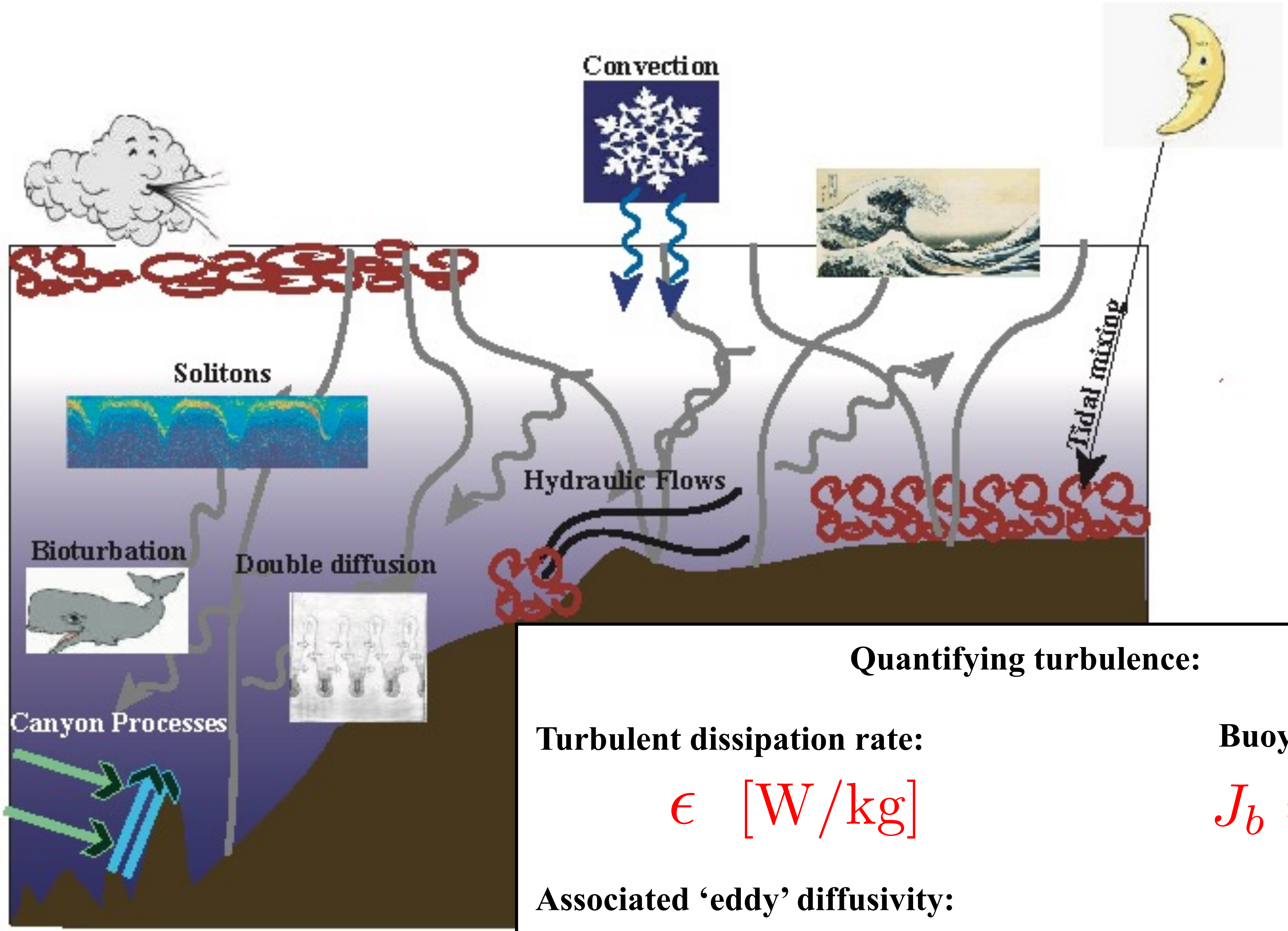
Global patterns of turbulence



The strength of small-scale turbulence varies by a factor of 1000! It has systematic patterns related to physical processes we are starting to understand

Whalen, Talley and MacKinnon, 2012
Whalen, MacKinnon, Talley, Waterhouse 2015

Diapycnal (vertical) Mixing Mechanisms



Quantifying turbulence:

Turbulent dissipation rate:

$$\epsilon \quad [\mathrm{W/kg}]$$

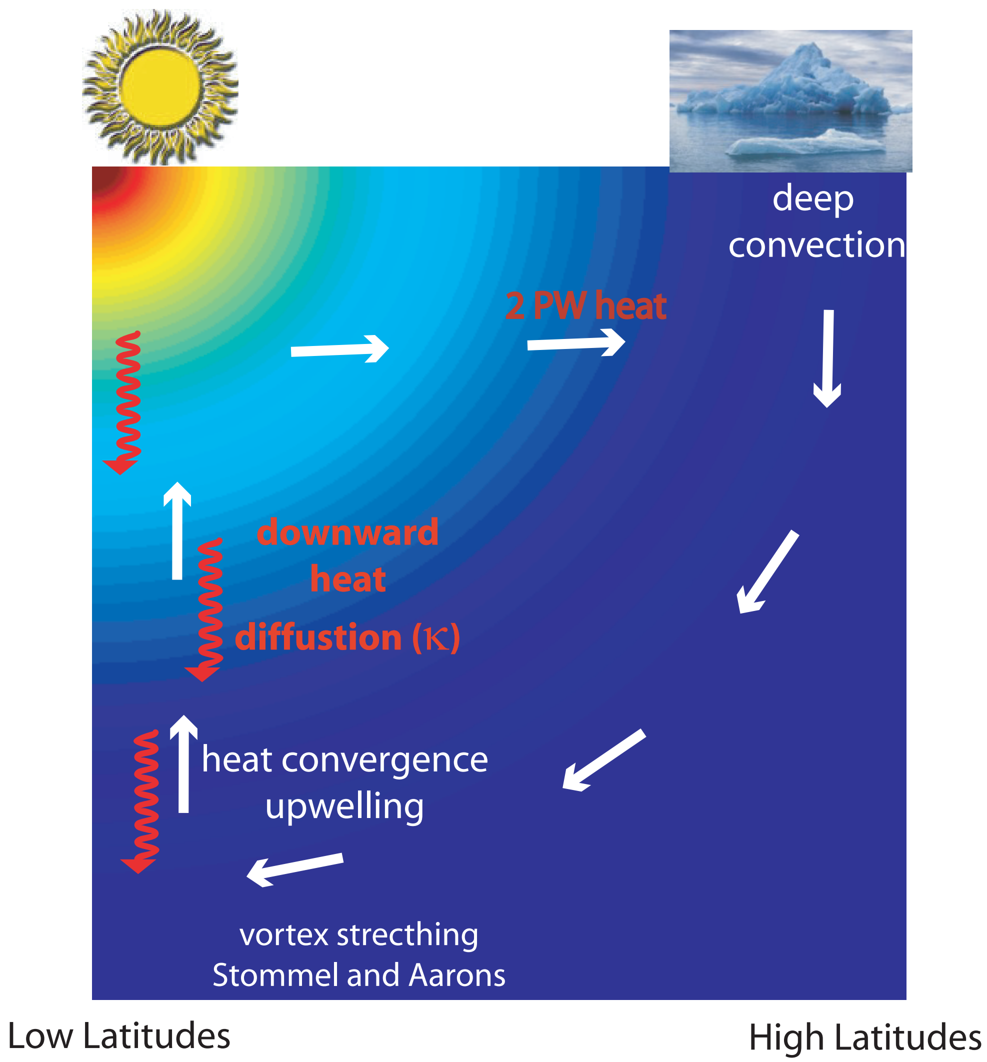
Buoyancy Flux:

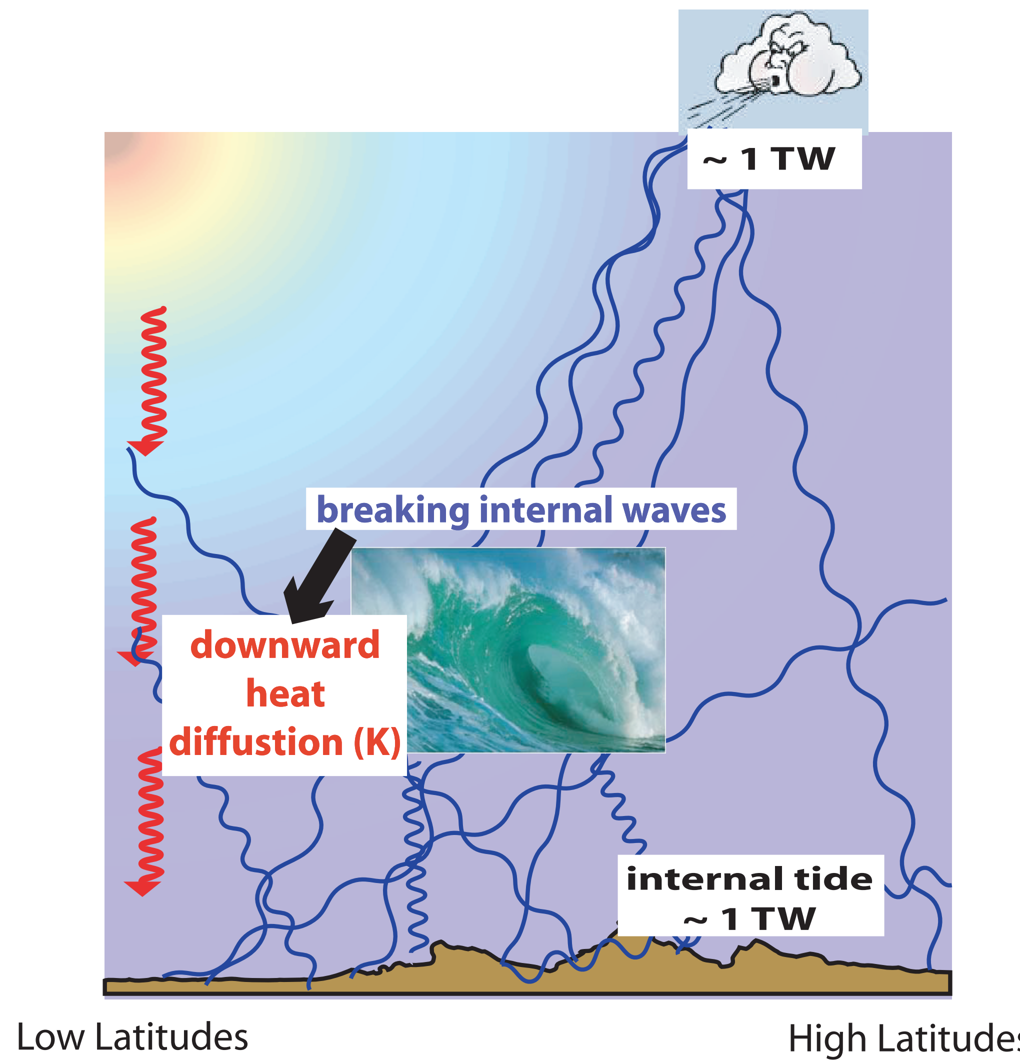
$$J_b = 0.2\epsilon$$

Associated 'eddy' diffusivity:

$$K_\rho = 0.2\frac{\epsilon}{N^2} \qquad [\mathrm{m^2/s}]$$

Turbulent mixing makes the ocean go round

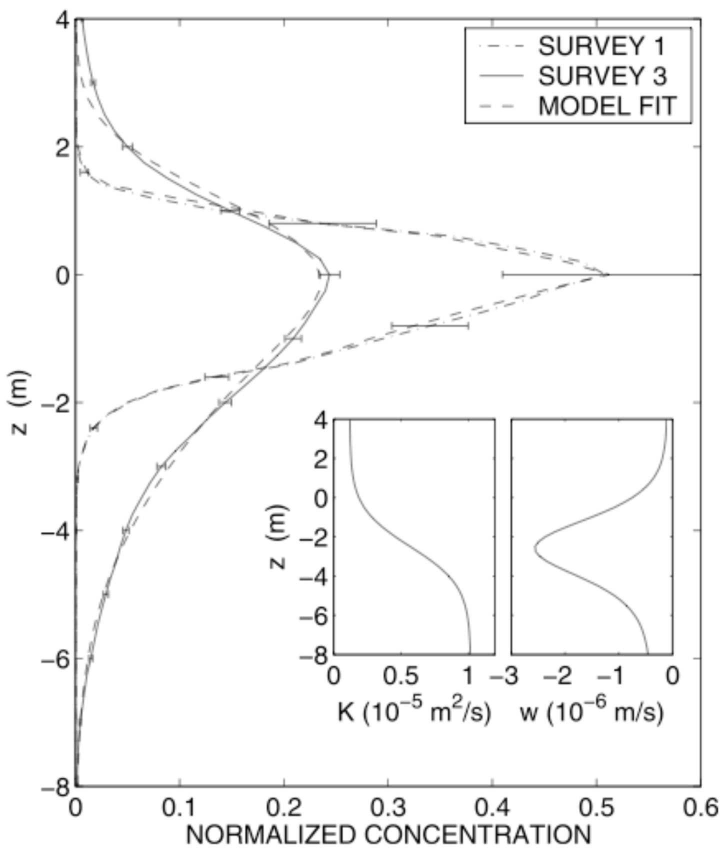




- Determines large scale vertical transport of heat, C02, nutrients, etc.
- Drives meridional overturning circulation by creating potential energy.

Measuring turbulent mixing

"Direct" method: dye dispersal (Ledwell et al., 2004)



Assume $K_\rho \sim K_{dye}$

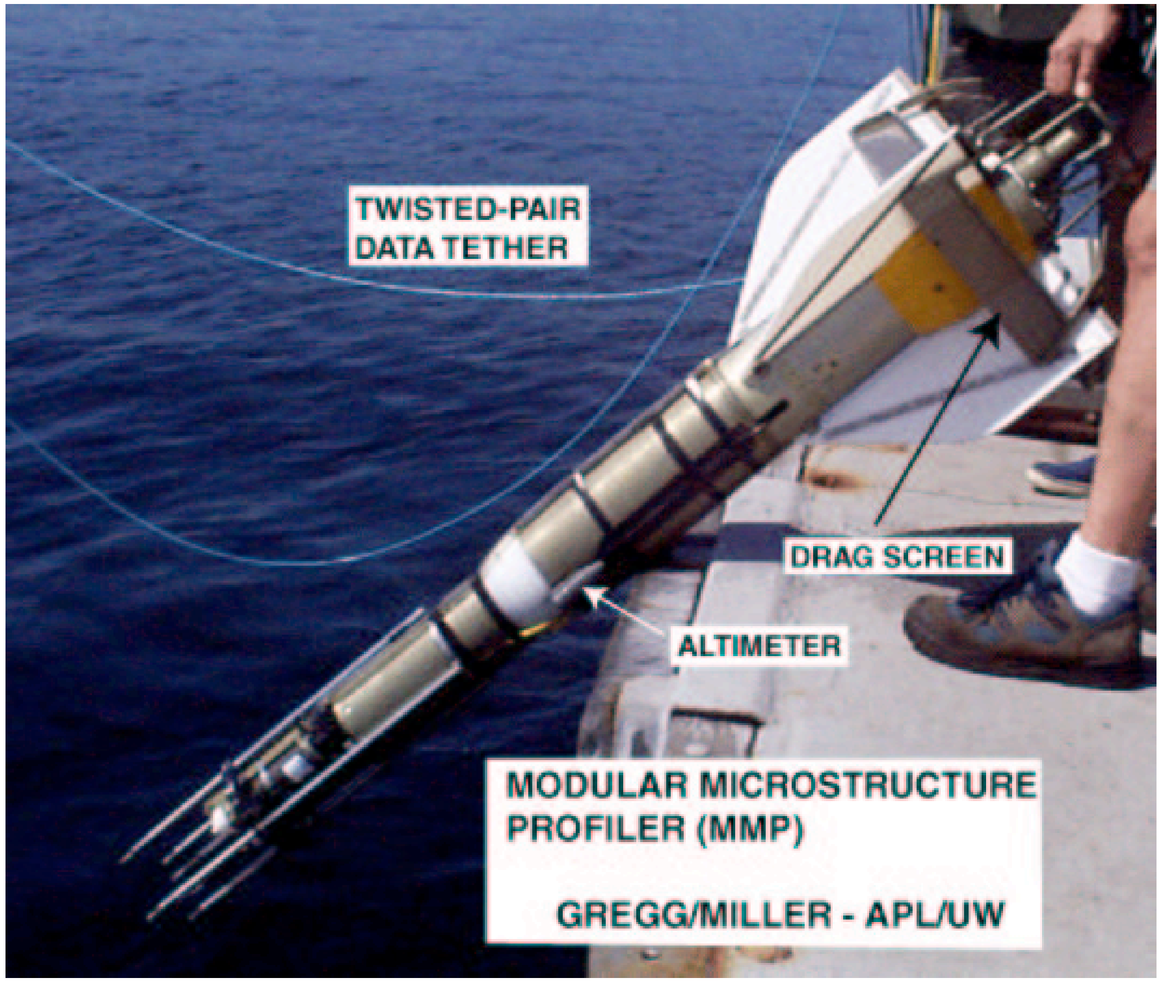
Indirect method 1: measure turbulence, assume mixing follows

$$K_\rho \approx 0.2 \frac{\epsilon}{N^2}$$

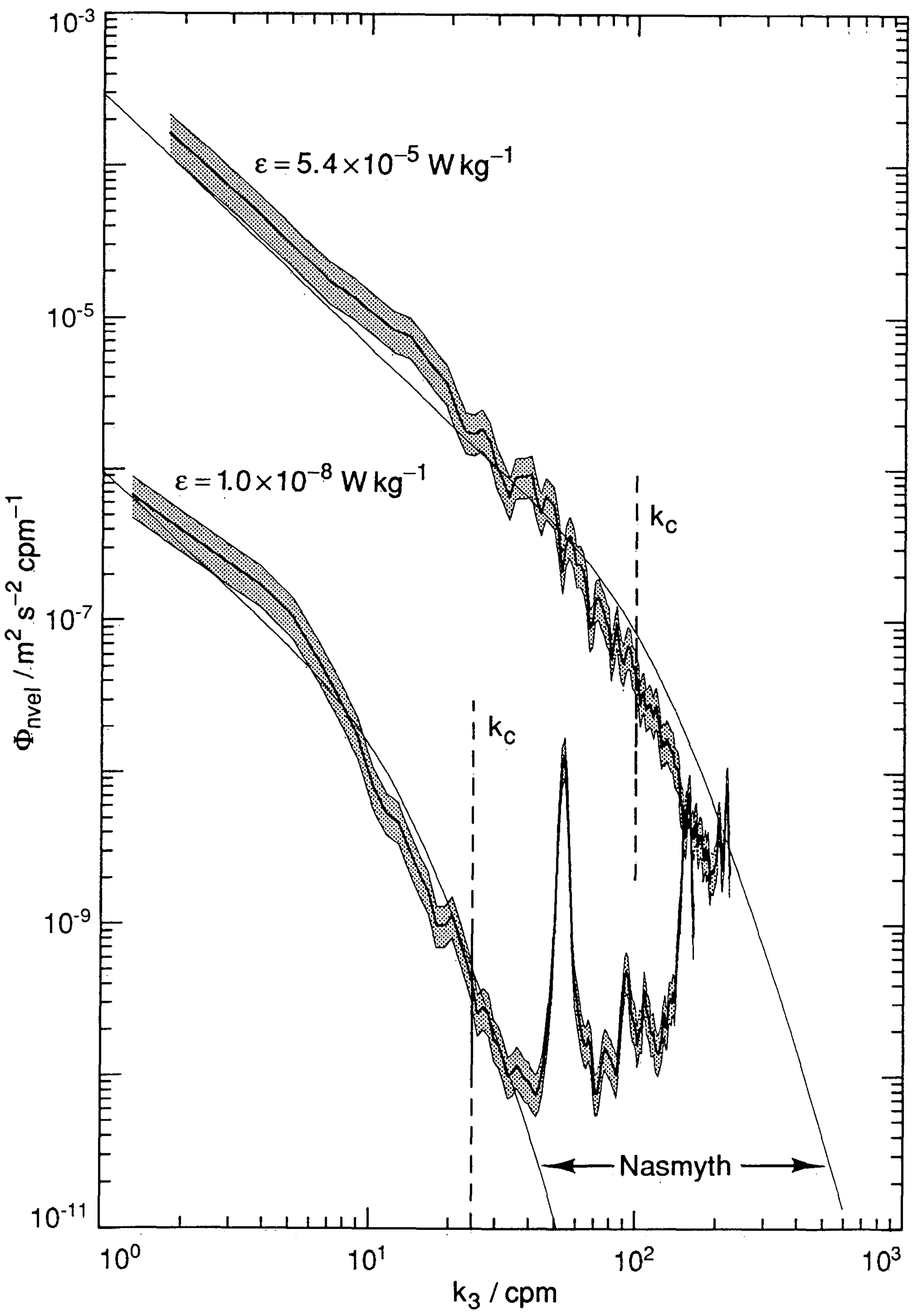
turbulent dissipation rate

(Osborn, 1980)

Measuring mixing: microstructure



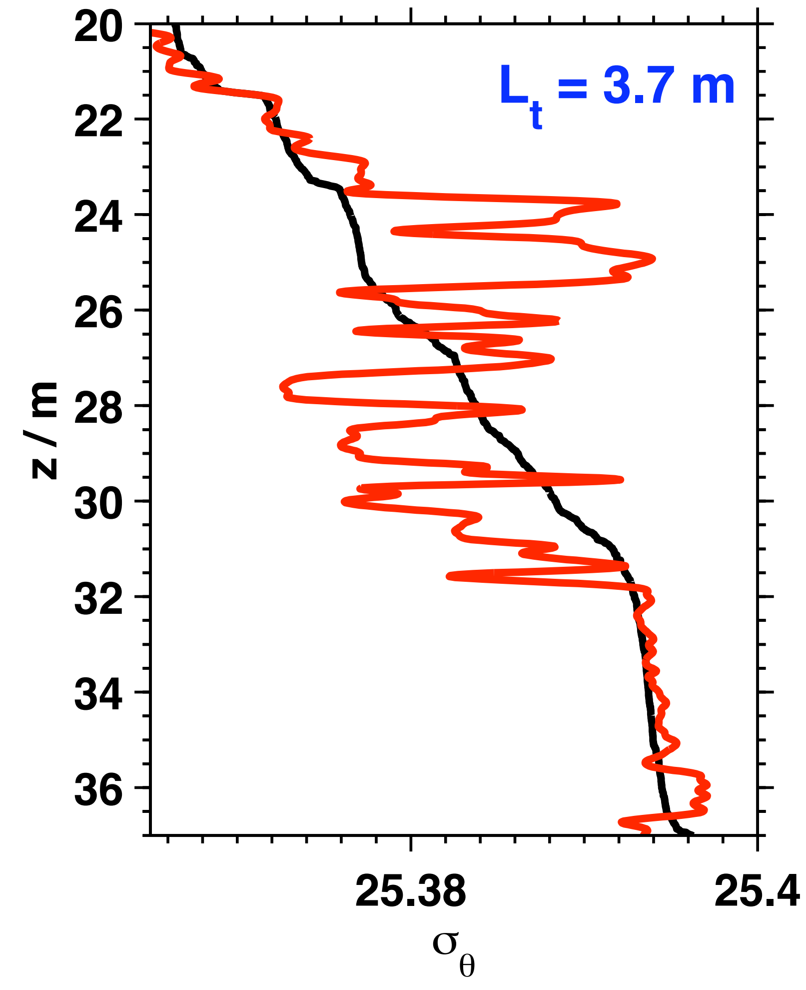
Measure inertial subrange of turbulence, either with velocity probes or fast thermistors. Fit turbulence model to estimate turbulent dissipation rate (ϵ)



(Wesson and Gregg 94)

Measuring mixing: inference

Next largest scale - cant see full turbulent cascasde, but maybe the ‘outer scales’ of turbulence. The **Thorpe scale** (Lt) estimates the size of an overturn.



$$\epsilon = \frac{\text{Eddy energy}}{\text{overturning time}} = L_t^2 N^3$$

This type of measurement can often be made with CTD data = mixing for the masses

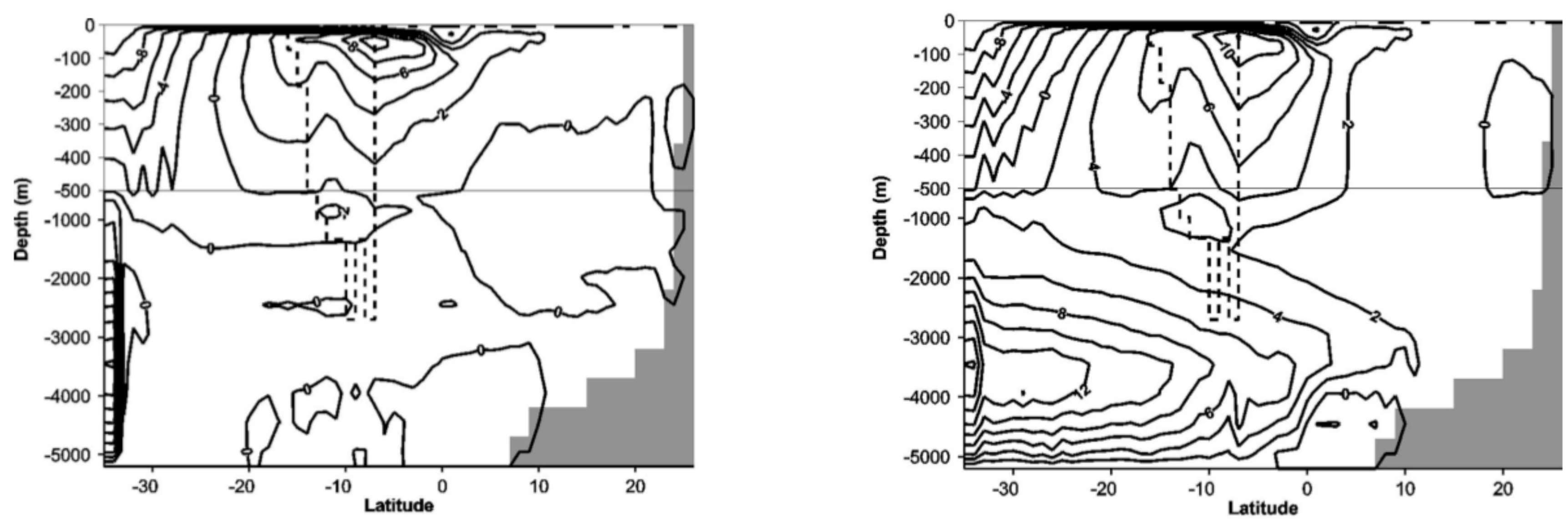
Patchy mixing matters

Palmer et al. (2007):

bottom-enhanced diffusivity

=> deep overturning

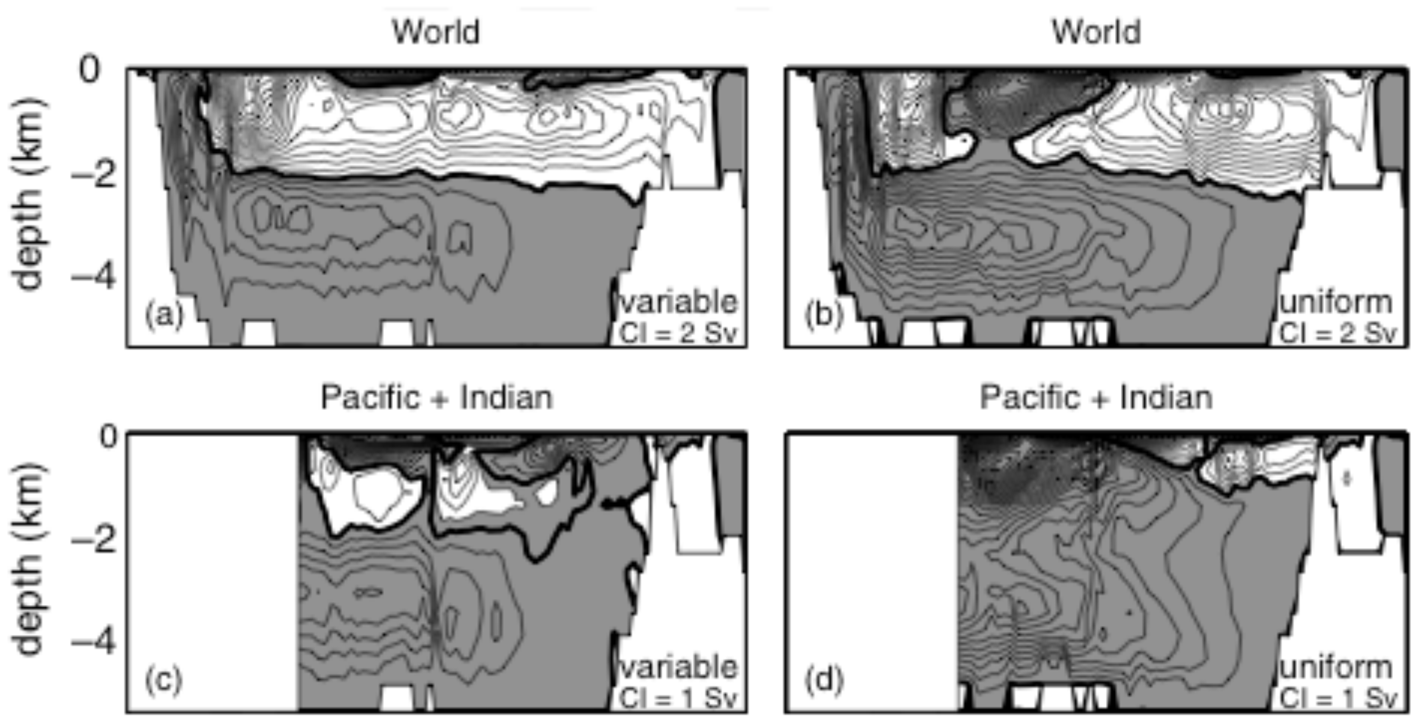
Constant κ = 1.2 10^{-4} | Bottom-enhanced diffusivity



Simmons et al. (2003):

enhanced mixing over rough topography

=> change in global MOC

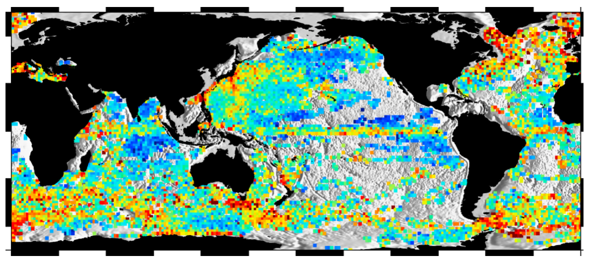


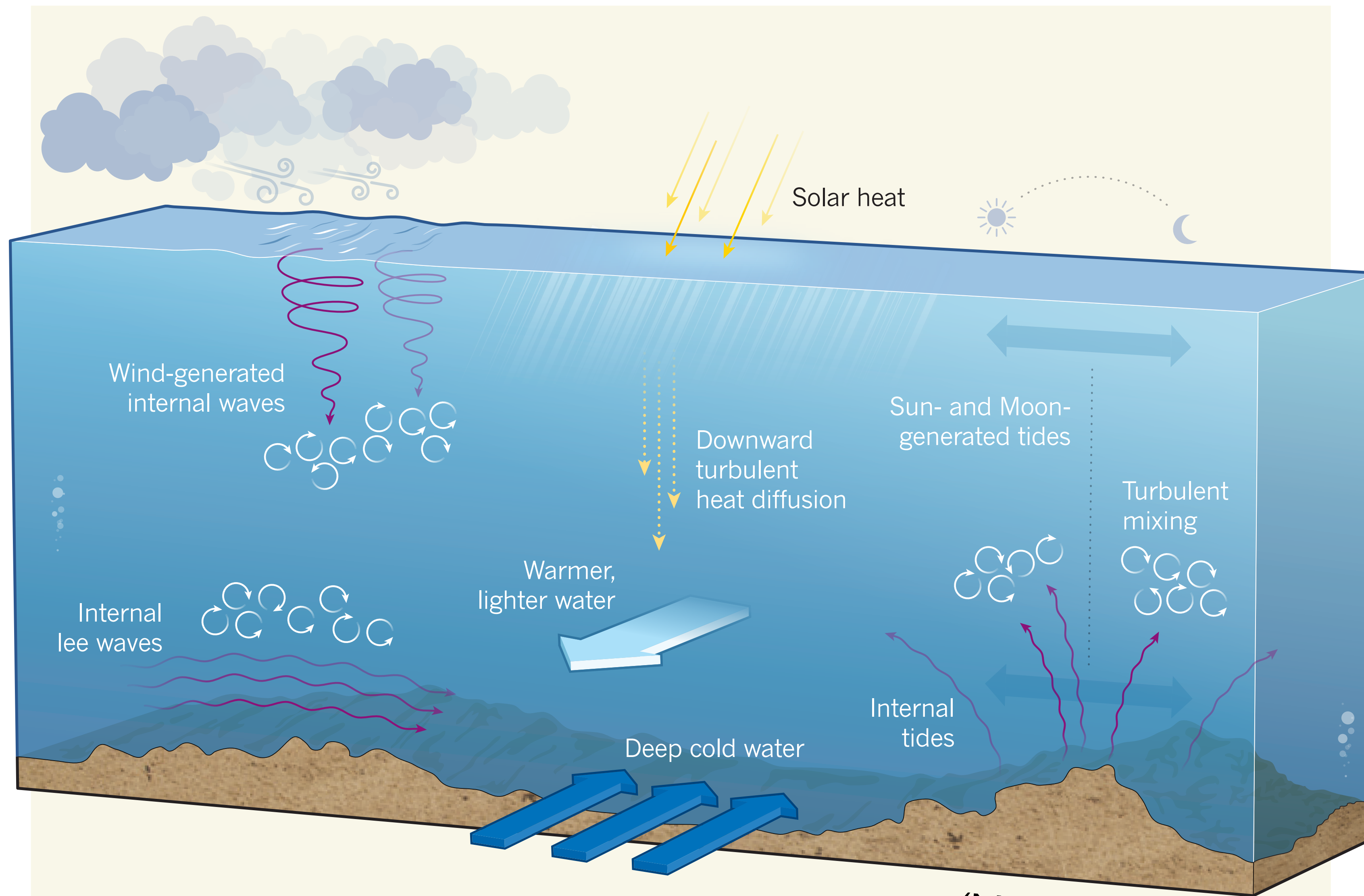
bottom-enhanced, spatially-variable mixing | uniform mixing

also: Hasumi & Suginohara (1999), Huang (1999), Katsman (2006), Saenko (2006), Jochum (2009)

Climate Process Team, 2010-2015

Our task: use what we collectively know about internal wave physics to develop a dynamic parameterization of diapycnal mixing that captures global patterns properly so that they can evolve in a changing climate.



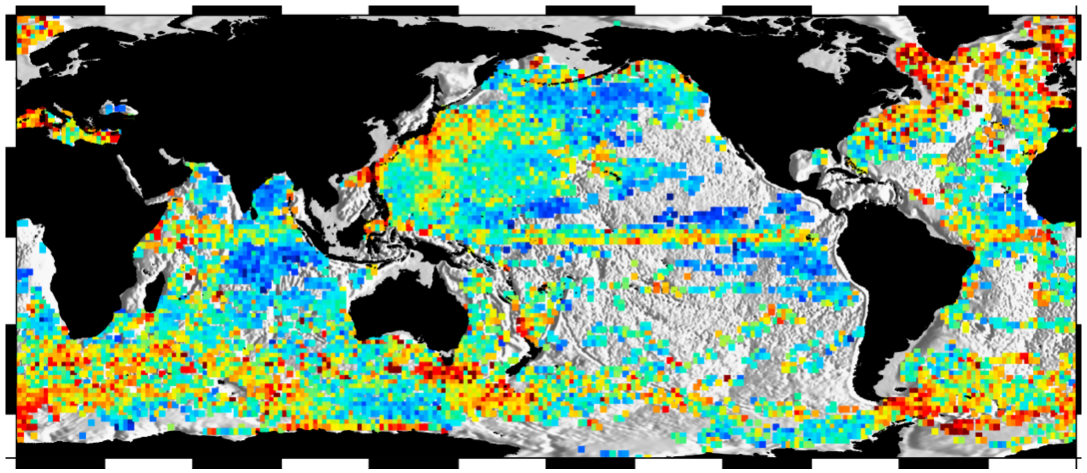


(MacKinnon 13)

Parameterizations of internal wave driven mixing will be available for general community use (https://github.com/CVMix)

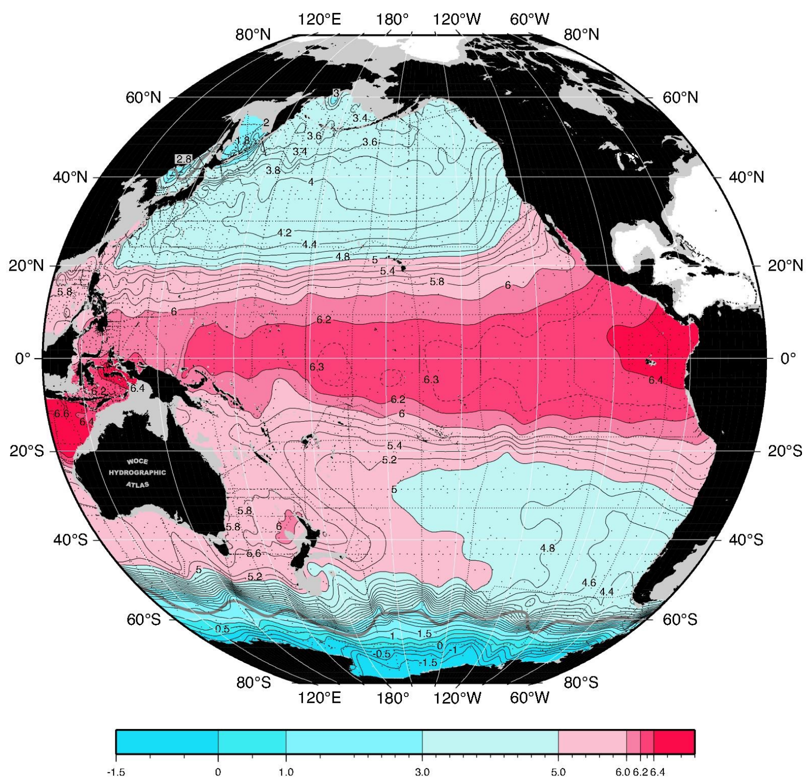
BAMS article, December 17

Convolving patterns

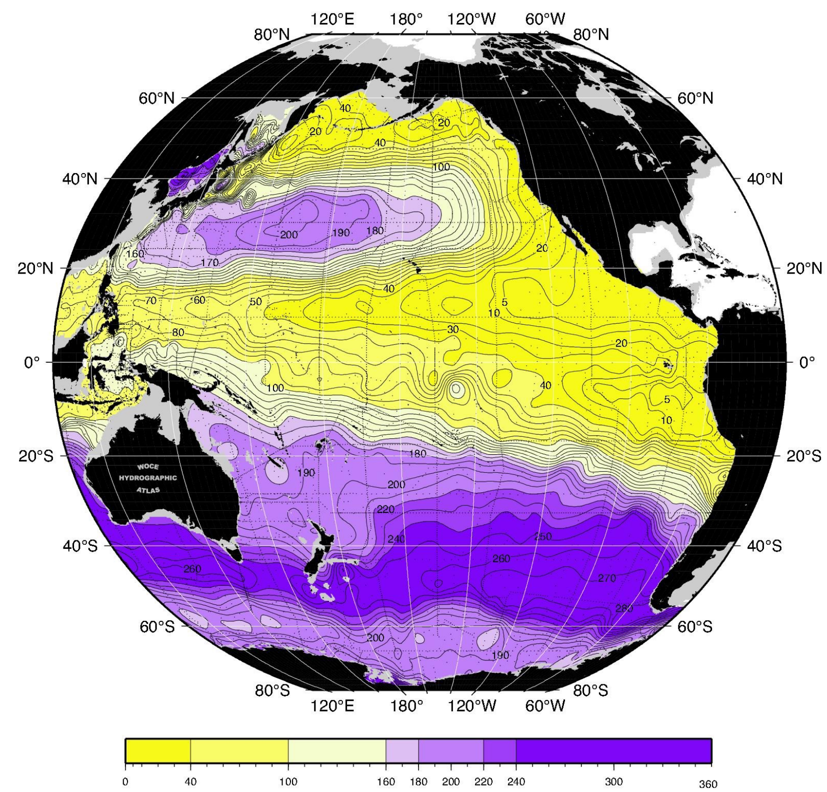


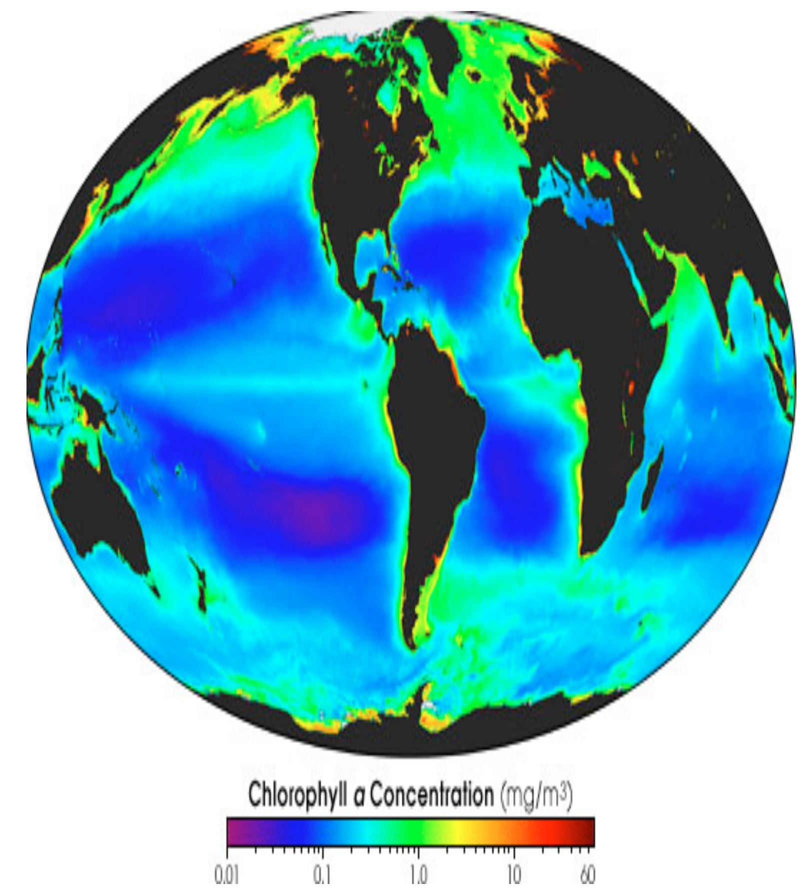
X

Potential temperature (ºC) 27.3 γ^n (kg/m^3)



Oxygen (μmol/kg) 500 m





= ??

Lateral mixing

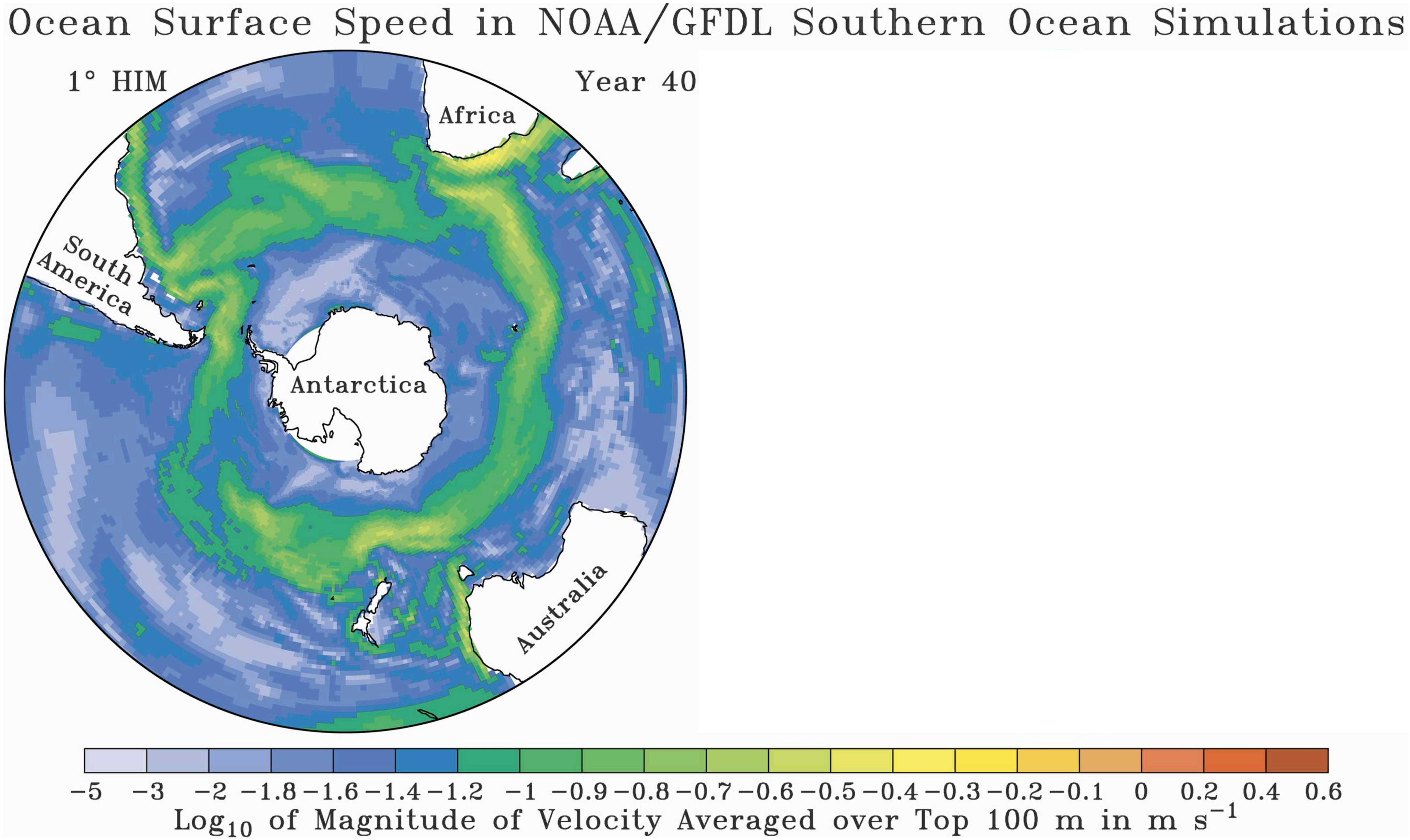


FIG. 6. Instantaneous surface speed in 1° and ⅙° models after 40 yr. Note that the large-scale structure of the 1° model is quite similar to the ⅙° model (the currents have similar locations and have similar horizontal extents). The main difference is in the presence of intense jets and eddies in the ⅙° model.

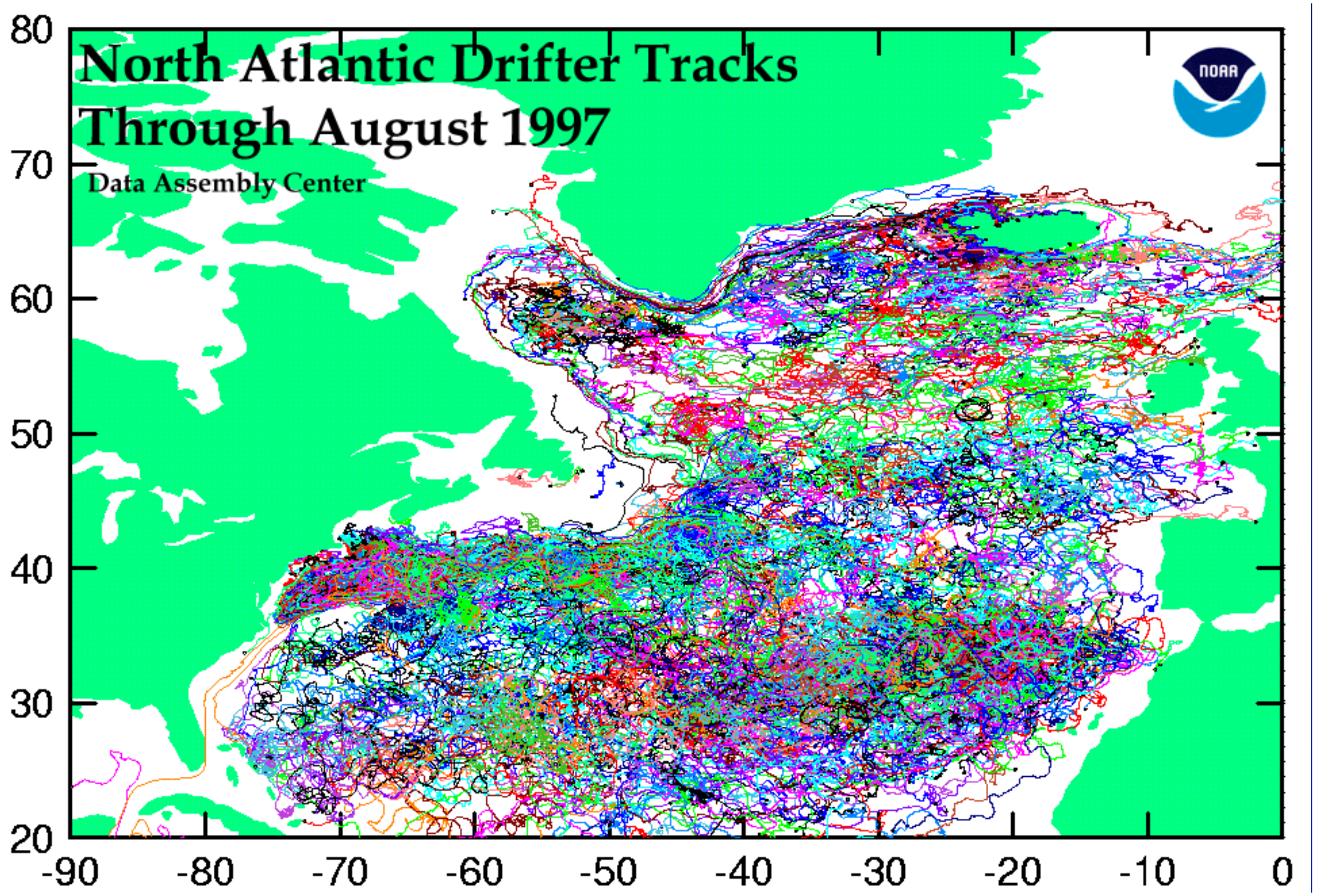
Lateral dispersion

eddy diffusivity

characteristic length scale

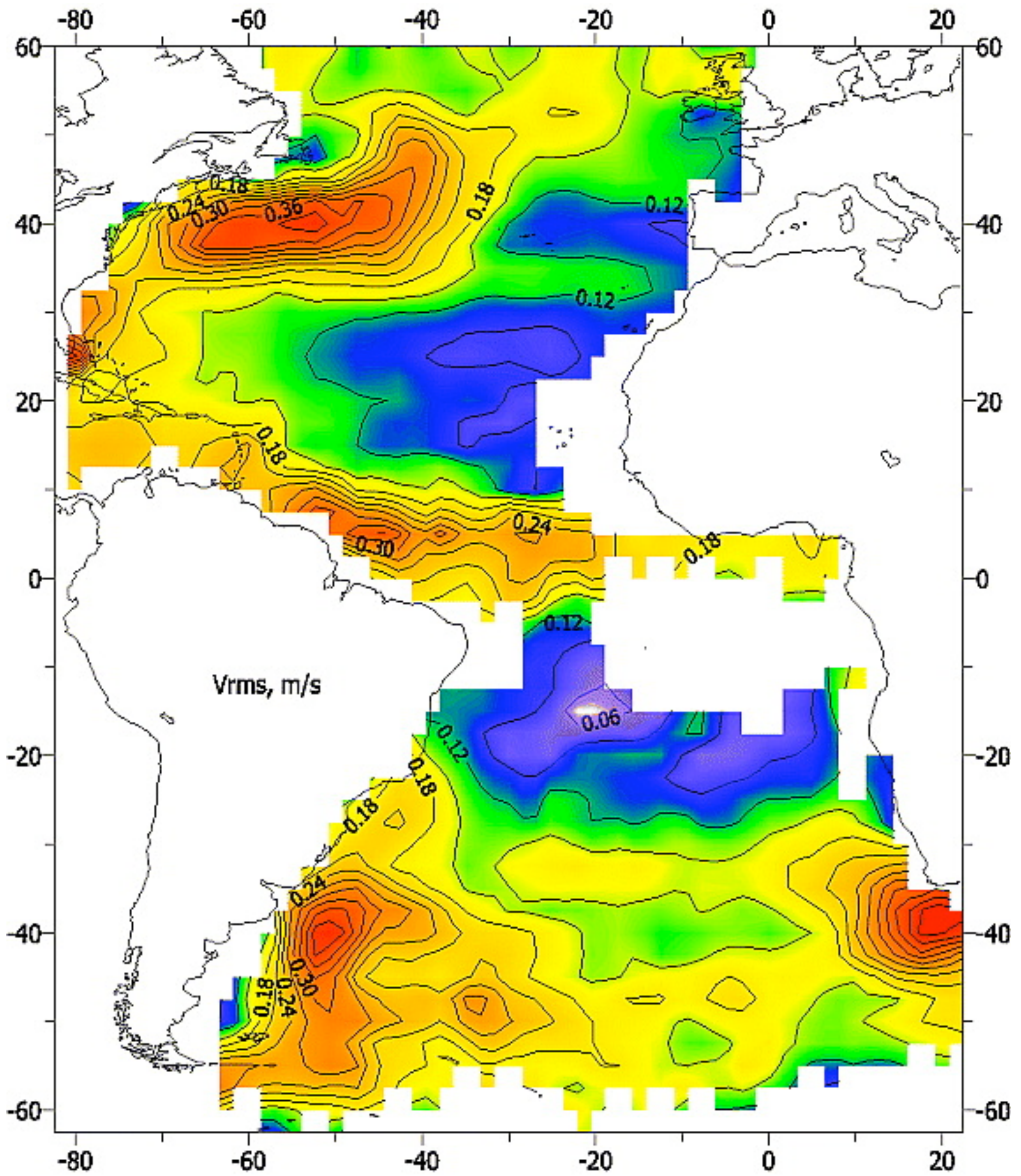
$$K \sim u\,L$$

characteristic velocity

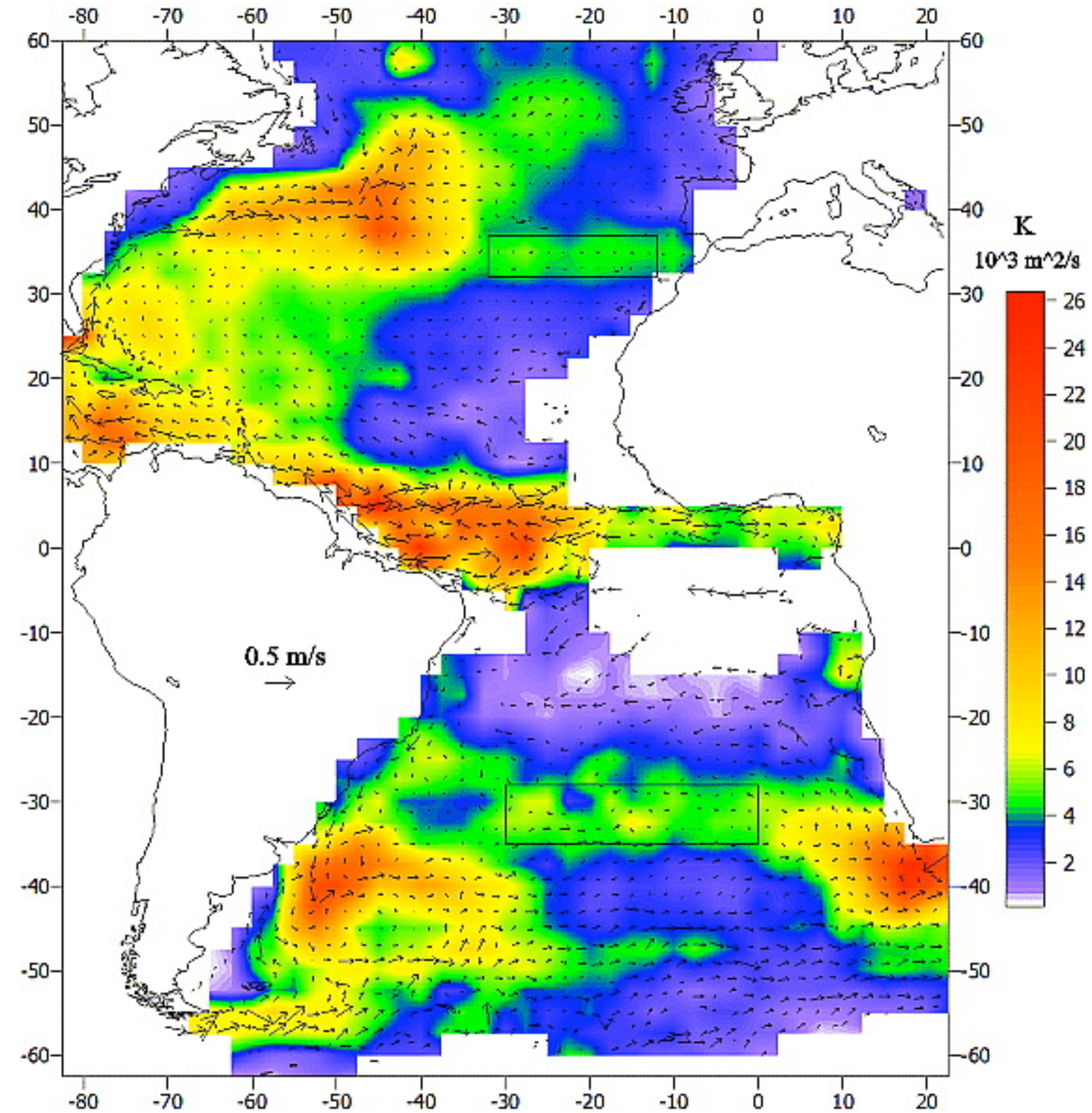


Lateral dispersion depends on strength/energy of eddies....

Eddy velocity scale $u \sim \sqrt{\mathrm{EKE}}$



Eddy diffusivity derived from drifter separations



Zhurbas and Oh 2004

Climate change uncertainties