

SIO 210 Introduction to Physical Oceanography  
Mid-term examination 11-12:30 PM; Eckart 227  
November 1, 2018; 1 hour 20 minutes

Closed book. (100 total points). One sheet (both sides) of your own notes is allowed. A simple calculator is allowed. No electronics with communications.

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**Possibly useful expressions and values**

$$1 \text{ Sv} = 1 \times 10^6 \text{ m}^3/\text{sec}$$

$$1^\circ \text{ latitude} = 111 \text{ km}$$

$$2\pi R$$

$$Ro = f/T = f/UL$$

$$g = 9.8 \text{ m/s}^2$$

$$\rho c_p T$$

$$\rho = 1025 \text{ kg/m}^3$$

$$c_p = 4000 \text{ J/kg}^\circ\text{C}$$

$$1 \text{ PW} = 10^{15} \text{ W}$$

$$F \sim \rho V(S_o - S_i)/S_m$$

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**Multiple choice** (10 problems, 2 points each, 20 points total)

For each problem, **circle the CORRECT answer**. (There should be **only one**.)

1. The volume transport of the Gulf Stream is approximately

- (a) 100 m/sec
- (b)  $100 \times 10^6 \text{ m}^3/\text{sec}$
- (c) 1 Sverdrup
- (d) 100 km x 5000 m

2. Which of the following components of the ocean-atmosphere heat flux is always into the ocean (warming the ocean)?

- (a) Sensible
- (b) Short wave
- (c) Latent
- (d) Long wave

3. If you sample sea surface height (SSH) once per day for 360 days, and the integral time scale of the SSH is 20 days, the number of degrees of freedom in your estimate of its frequency spectrum is

- (a) 1
- (b) 360
- (c) 20
- (d) 18

4. Which of these makes Lagrangian observations?

- (a) Satellite ocean color sensor
- (b) Moored current meter at the equator
- (c) Research ship making CTD observations every 100 km between California and Asia
- (d) **Subsurface float in the Argo program**

5. The pressure gradient force
- (a) Points towards the center of the Earth
  - (b) Always results in acceleration of the water
  - (c) **Points from high pressure to low pressure**
  - (d) Is almost always balanced by friction

6. The moon causes semidiurnal tides (period about 12 hr) and diurnal tides (period about 24 hr) in the ocean because the earth rotates on its axis once every 24 hours. If the earth rotated on its axis once every 48 hours, the periods of the lunar tides would be about

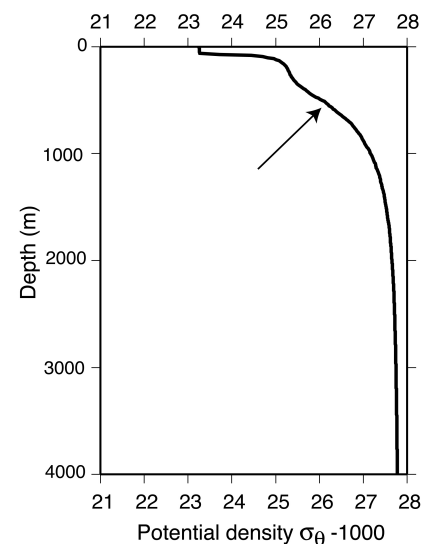
- (a) 6 and 12 h
- (b) 12 and 24 h
- (c) **24 and 48 h.**

7. In hydrostatic balance

- (a) Vertical acceleration is non-zero.
- (b) Coriolis force balances the pressure gradient force
- (c) **Gravity balances the pressure gradient force.**
- (d) Pressure depends on water velocity.

8. In the vertical profile of density shown in the figure, the arrow points to the

- (a) Thermocline
- (b) Mode water
- (c) Abyssal ocean
- (d) **Pycnocline**



9. In the Mediterranean Sea there is excess evaporation and cooling of waters that flow in from the Atlantic Ocean through the Strait of Gibraltar. These waters then return to the Atlantic Ocean. Flow through the strait is characterized by the following

- (a) Potential density of the outflow is less than the potential density of the inflow
- (b) **Salinity of the inflow is less than the salinity of the outflow**
- (c) Mass transports into the Mediterranean is less than the mass transport out of the Mediterranean.

10. Sound speed

- (a) increases as temperature becomes colder for a fixed pressure
  - (b) has a minimum at the bottom in the deep ocean (> 4000 m depth)
  - (c) increases as pressure becomes higher for a fixed temperature
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**Short answer or calculations (80 points)**

**11. (10 points)**

The Hovmöller diagram shows observed sea level anomaly at the equator in the Pacific Ocean.

- (a) Describe what a Hovmöller diagram is and why it can be useful.

A Hovmöller diagram contours a property in time and space. In this case, the space axis is longitude, and time axis is years/months. It is useful for displaying the spatial structure of the time series. It can be used to demonstrate propagation of features.

- (b) How was the information in this diagram collected? (what instrument?) If you don't really know, then make some good guesses and explain your answer.

This data was collected using a satellite altimeter. (extra information: The property that is displayed is the sea level anomaly, because the absolute distance of the altimeter to the sea surface includes geoidal distances that have nothing to do with the dynamical ocean.)

I accepted answers that included network of profiling buoys (like Argo). I would have accepted answers like equatorial array of moorings, but did not accept answers with a single mooring, because a Hovmuller diagram along the equator cannot be produced from a single mooring.

- (c) If the sea level is given by  $h(x,t)$ , write an expression for the sea level 'anomaly'.

$$h'(x,t) = h(x,t) - \text{mean}(h(x,t))$$

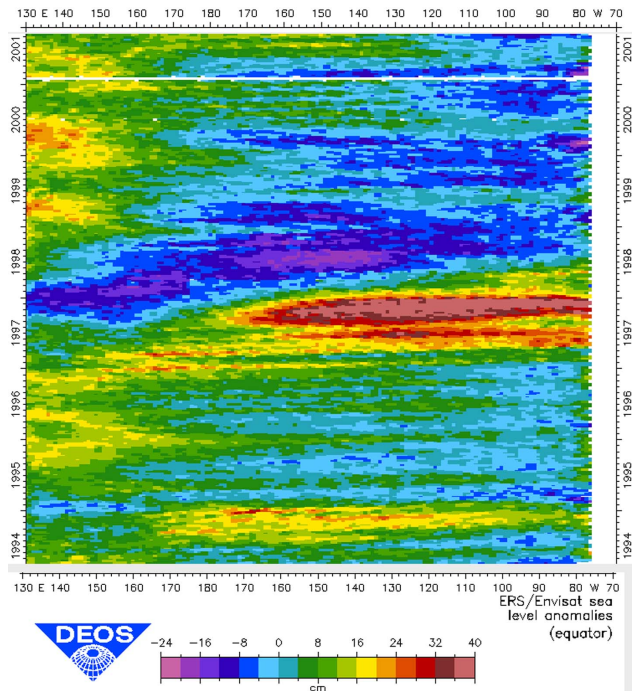
- (d) Describe in words the anomaly pattern during 1997 and 1998. What do you see, is it propagating? If so, what is the approximate speed of propagation?

In 1997 there is an anomalously high sea level in the central and eastern Pacific. This is in the region of the usual 'cold tongue' that we've highlighted on some temperature maps. It is also a region of usually low surface height. This signal does not appear to propagate much in this diagram.

In 1998 a region of anomalously low sea level develops in the western Pacific and propagates eastward, at which point it appears at a range of central and eastern longitudes. This particular event is strongest in the western and central Pacific, not symmetric with the high sea level in 1997.

There is some propagation in the western Pacific: about 35 degrees longitude over 6 months. If you calculate the speed:  $35 \times 111 \text{ km} \sim 3500 \text{ km}$  in 6 months =  $1.55 \times 10^7 \text{ sec}$ . The corresponding speed is  $3.5 \times 10^6 \text{ m} / 1.55 \times 10^7 \text{ sec} = .22 \text{ m/sec} = 20 \text{ cm/sec}$  which is a reasonable propagation speed for the ocean.

- (e) Extra credit: do you know the name of the phenomenon that caused the 1997-1998 anomalies? El Nino in 1997 and La Nina in 1998, or in general, El Nino Southern Oscillation (ENSO)



**12. (20 points)**

(a) What is salinity? (brief answer)

Salinity is the ratio of mass of dissolved matter (mostly salts) to the total mass of seawater. It is usually expressed in grams/kilogram, although for actual calculations, this should be converted to kg/kg.

(b) What is the approximate range of salinity in the world ocean (not including very near-shore regions affected by local river runoff etc)? \_\_\_\_\_

30 to 40 is the maximum range. General open ocean might be more like 32 – 38. I will accept any reasonable range as an answer.

The figure is a salinity section from south to north through the length of the Atlantic Ocean.

c) Circle any region of high salinity near the UPPER surface of the ocean. Given an explanation in terms of atmospheric forcing of why salinity is high in the region you've circled.

Circle high salinity near 30°N or 20°S, or both.

In both regions, the atmospheric circulation is dominated by the subtropical highs, where air sinks and is very dry. There is net evaporation in these regions, so salinity is high.

(d) Why is it possible to have high salinity water near the sea surface? (Why doesn't it sink?)

The surface water has to be less dense. Therefore it is significantly warmer than underlying water, which supports the higher salinity surface. (extra information: this can lead to interesting local convection on daily or seasonal time scales as the surface water cools and there might be enhanced overturn due to higher salinity. We have not mentioned double diffusion in class, but when there is warm/salty water

over cooler/fresher water, it can vertically mix in an interesting way as temperature and salinity have different molecular diffusivities that somehow scale up to the macroscale that we observe.)

(e) On the blank potential temperature/salinity diagram (on the next age):

(i) mark where your circled high salinity water might be found. Then

(ii) mark a place where a lower salinity BOTTOM water parcel might be found.

For (i), I picked water in the northern hemisphere high salinity feature, of 37 psu, which has a temperature of about 20°C. The properties are pretty similar in the southern hemisphere feature. (You could guess at this temperature, or notice that I used the same section for the last question. There you would see that the temperature at the high salinity is about 20°C.)

For (ii), I picked bottom water directly beneath this. For the NH that would be around 34.8 psu and 1.5°C. For the SH that would be 34.7 psu and 0.5°C.

(f) The contours in the diagram are ‘potential density’ relative to the sea surface, which we call either  $\sigma_\theta$  or  $\sigma_0$ . What are the potential densities  $\sigma_\theta$  of the two parcels you marked in (e)?

NH choices: 26.0 and 27.8

SH choices: 26.0 and also about 27.8

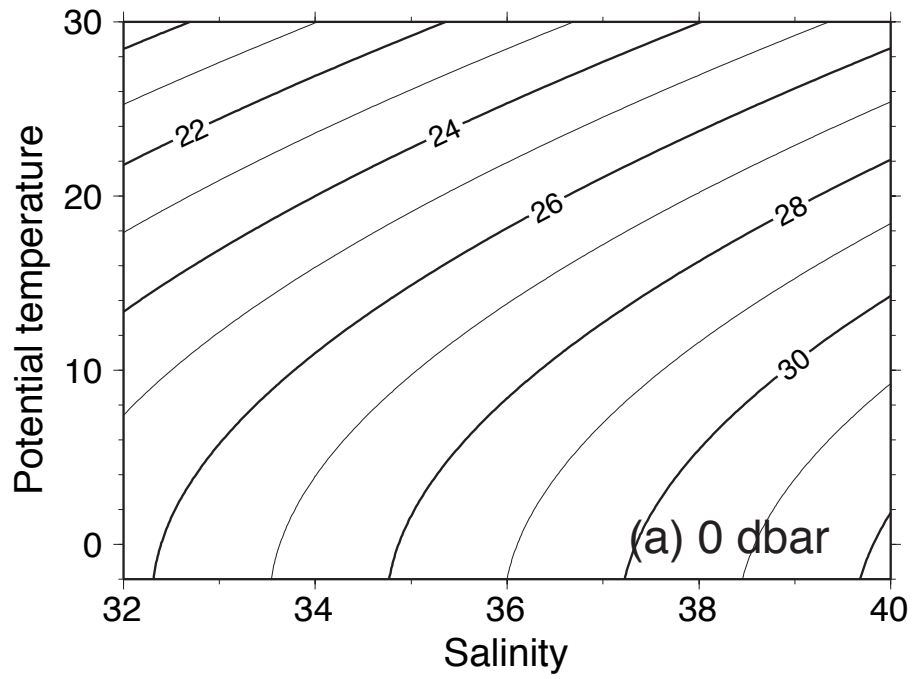
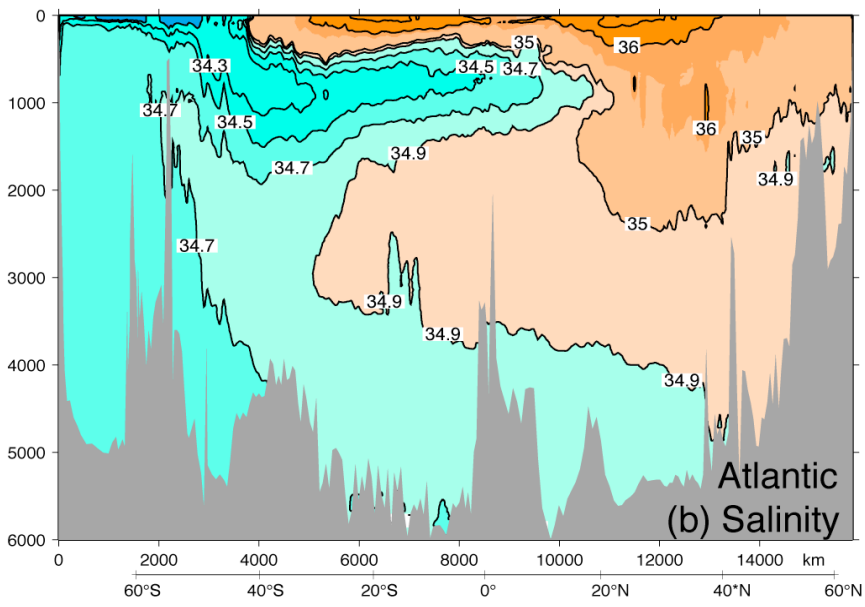
(It’s hard to distinguish between the densities of the two deep waters on this theta-S diagram.)

(g) On this potential temperature/salinity diagram, sketch the contours of potential density  $\sigma_4$  relative to 4000 dbar. This will not be exact. I am looking for the relative slope of your  $\sigma_4$  contours compared with  $\sigma_0$ .

Draw contours that are flattened relative to the  $\sigma_\theta$  contours.

(h) The difference in contouring for potential density  $\sigma_\theta$  and potential density  $\sigma_4$  occurs (mostly) because the compressibility of seawater depends on temperature. Describe this dependence and how it leads to the different slopes of  $\sigma_\theta$  and  $\sigma_4$ .

The compressibility depends on temperature. (It also depends on salinity, but the dependence is much weaker. Warm water is less compressible, cold water is more compressible. We can start by comparing water parcels with each other at the sea surface, using  $\sigma_\theta$ . Then when we compare them at great depth, their relative density difference will have changed. The cooler water parcel will compress more, and become even denser relative to the warmer water parcel. This is best illustrated if the two parcels have the same density at the surface pressure (or at the bottom pressure).



**13. (12 points)**

The molecular diffusivity of temperature is

$$\kappa_T = 0.0014 \text{ cm}^2/\text{sec} = 1.4 \times 10^{-3} \text{ cm}^2/\text{sec} = 1.4 \times 10^{-7} \text{ m}^2/\text{sec}$$

(a) Using just this diffusivity, estimate the time scale for molecular diffusion over a length of 1 km.

Use the units for diffusivity to make this calculation, no need for a more complicated expression.

$$1 \text{ km} = 10^3 \text{ m}$$

$$\text{Time} = (\text{Length})^2/\kappa_T = (10^6 \text{ m}^2)/(1.4 \times 10^{-7} \text{ m}^2/\text{sec}) = .71 \times 10^{13} \text{ sec}$$

$$\text{This can be converted to years: } (60 \text{ sec}/\text{min})(60 \text{ min}/\text{hr})(24 \text{ hr}/\text{day})(365 \text{ day}/\text{yr}) = 3.15 \times 10^7 \text{ sec}/\text{yr}$$

$$\text{Time} = .71 \times 10^{13} \text{ sec}/(3.15 \times 10^7 \text{ sec}/\text{yr}) = 225139 \text{ years}$$

Horizontal eddy diffusivity can be

$$A_T = 1 \times 10^5 \text{ cm}^2/\text{sec}$$

(b) Estimate the time scale for eddy diffusion of temperature over a length of 1 km. (Be careful about units.)

Same calculation, replace eddy diffusivity

$$A_T = 1 \times 10^5 \text{ cm}^2/\text{sec} = 10 \text{ m}^2/\text{sec}$$

$$\text{Time} = (\text{Length})^2/A_T = (10^6 \text{ m}^2)/(10 \text{ m}^2/\text{sec}) = 1 \times 10^5 \text{ sec}$$

Convert to hours, days, years

(c) If the property being diffused is salt, are the molecular and eddy diffusivities the same as for temperature? Answer for each of the two (molecular and eddy).

For molecular diffusivity, the temperature and salinity diffusivities differ because these are very different properties and behave differently.

For eddy diffusivity, temperature and salinity diffusivities are the same because they are both mixed by the same turbulent field.

(d) What processes might contribute to the horizontal eddy diffusivity?

Eddy diffusivity refers to mixing due to any length scale that is smaller than the length scale of the problem. For the large-scale circulation, horizontal eddy diffusivity would be due to lateral stirring by the mesoscale and submesoscale. (Note that vertical eddy diffusivity is different and is mostly due to overturns driven by breaking internal waves.)

**14. (12 points)**

A wave is characterized by a wavelength and period.

(a) Sketch a wave in spatial coordinates (x) and show the wavelength  $\lambda$ . Label the crest and trough.

Everyone answered this correctly unless they forgot to read the whole question (i.e. to label the crest and trough). (Won't sketch in word document.)

(b) Sketch a wave in time coordinates (t) and show the period T. How is the period measured at a given location?

Almost everyone got this as well. (Won't sketch in word document.)

The period is measured by observing the height of the wave or velocities within the wave at a fixed point, and measuring between successive troughs or crests. Types of measurements could be a wave staff, or marker on a pier, or velocity measurement on a buoy, etc.

(c) What is the expression for frequency f in terms of the period T?

$f = 1/T$  or  $f = 2\pi/T$  depending on context. (Either answer is acceptable.)

(d) What is a typical wavelength for a wind wave? (surface gravity wave driven by wind)

Typical wavelength can be 1 to 150 m.

(e) In water depth of  $H = 100$  m, using your answer for (d), which dispersion relation does your wave obey:

$$c^2 = (gH) \text{ or } c^2 = (g\lambda/(2\pi))?$$

If the wavelength is much shorter than 100 m, then choose the second expression that does not include water depth. If it is comparable to or longer than 100 m, choose the first expression that does include water depth H.

(f) Based on your answer to (e), is your wave a deep water wave or a shallow water wave? If the wavelength is much shorter than 100 m, then it is a deep water wave. If it is comparable to or longer than 100 m, it is a shallow water wave

### 15. (10 points)

The momentum equations in a fluid in a non-rotating reference frame are

$$x: \partial u/\partial t + u \partial u/\partial x + v \partial u/\partial y + w \partial u/\partial z = - (1/\rho)\partial p/\partial x + \partial/\partial x(A_H \partial u/\partial x) + \partial/\partial y(A_H \partial u/\partial y) + \partial/\partial z(A_V \partial u/\partial z)$$

$$y: \partial v/\partial t + u \partial v/\partial x + v \partial v/\partial y + w \partial v/\partial z = - (1/\rho)\partial p/\partial y + \partial/\partial x(A_H \partial v/\partial x) + \partial/\partial y(A_H \partial v/\partial y) + \partial/\partial z(A_V \partial v/\partial z)$$

$$z: \partial w/\partial t + u \partial w/\partial x + v \partial w/\partial y + w \partial w/\partial z = - (1/\rho)\partial p/\partial z - g + \partial/\partial x(A_H \partial w/\partial x) + \partial/\partial y(A_H \partial w/\partial y) + \partial/\partial z(A_V \partial w/\partial z)$$

In the following you can refer to either the 'word equation' or the actual equations

(a) Which pair of terms yields hydrostatic balance? (circle and label) z momentum equation, yellow highlighted terms.



(b) Circle the advective terms in the x momentum equation. Describe in words how one of these terms works (what is the process for changing momentum using this term).

Yellow highlighted terms in x momentum equation. I was looking for an explanation in terms of how advection works: if there is difference in the velocity from one location to another say, in y, then the velocity field itself will move the higher or lower velocity to the new location. This causes a change in the velocity at the measurement point.

c) Circle and label the viscous terms in the y momentum equation. Describe briefly how these terms work (what is the process for changing momentum using this term).

Yellow highlighted terms in y momentum equation. Diffusion by either molecules or by turbulent structures in the flow, exchanges particles. If there is a velocity shear at the exchange location, then the exchange will exchange the velocities themselves. This is Fick's Law: that this exchange creates a flux, in this case of momentum. If there is a convergence of the flux, then the velocity itself will change.

### 16. (16 points)

On the attached color sections,

- (a) Circle and label the North Atlantic Deep Water (NADW)
- (b) What property (ies) did you use to identify the NADW?

High salinity at 1000-3000 m depth, high oxygen.

- (c) Circle and label the Antarctic Intermediate Water (AAIW)
- (d) What property (ies) did you use to identify the AAIW?

Low salinity at about 500 m depth in southern hemisphere.

- (e) Circle and label the Mediterranean Water (MedW)
- (f) What property (ies) did you use to identify the MedW?

High salinity at about 1000 m depth in northern hemisphere. (Not extending to the sea surface, which is high salinity because of local evaporation.)

- (g) On the potential temperature section, at about 10-15°S, there is an inversion in temperature. Circle this inversion. If density depended only on temperature, there would be a density inversion here as well. Is it possible to have a density inversion in the ocean at this space/time scale? No, it is not. The space/time scale is hundreds of kilometers horizontally and hundreds of meters vertically, and 'permanent', i.e. part of the mean circulation with time scale of hundreds of years. \_\_\_\_\_ (I did accept answers of Yes if explained in terms of potential density, which is not all shown in this figure.)

Explain how this situation can be stable over hundreds of years. The temperature stratification is compensated by a strong, stable salinity stratification, with fresher water in the upper part of the temperature inversion (cooler) and saltier in the lower part of the inversion (warmer).

[I did accept answers that invoked an inversion in potential density relative to a different pressure than the depth of the temperature inversion, which would then appear to show a density inversion where none exists if the density is referenced to the local pressure at the center of the inversion.]

