# 210 Dynamical Concepts

1. This lecture a primer in fluid mechanics. All in words. Can't fully solve problems this way but can understand solved ones. This mode of thinking usually has to go before detailed analysis.

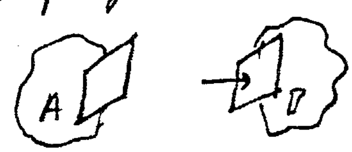
2. Motion of fluid parcels governed by $F = ma$. ~~[illegible]~~

   Given all forces, can calculate $a = du/dt$.
   We will catalog a number of forces, using $a = F/m$ to estimate the associated accelerations. Especially if $a = 0$, $\Sigma F = 0$

3. Thus **gravity** is a force / unit mass toward the earth's center. It results in an acceleration $g = 980\ cm/sec^2$ toward the center of the earth if no other forces act.

4. **Pressure Gradient Force**

   What is pressure? It is the F/area any volume of fluid exerts across its boundaries on any adjacent volume. Not assoc w. motion

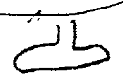



A pushes on B
B pushes on A

In air F/area ≡ pressure = $14.7\ lb/in^2$
= 1 atm
= 1000 mb Hg
= weight of $1\ cm^2 \times 10$ m water column.
= $10^6\ dy/cm^2$

> Thus I weigh 160 lb, area of soles of feet is 6x12 = 72 square inches.
> Hydrost. Prof. pressure = $160/72 = 2.2\ lb/in^2$
> Adj hydrostatic implies ∃ nonhydrostatic.
> Q How can $p \neq$ weight/area?
> A If in accelerated (jump) or in accelerated coordinates (elevator)
> Q Weight of air column = $14.7\ lb/in^2$
> Why don't bath scales say $14.7 \times$ area?

Is HP = weight of column? No



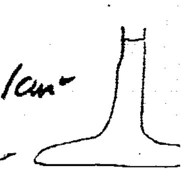
5. In ocean/atm pressure is hydrostatic. In ocean
   Pressure at depth = weight of $1\ cm^2$ cross section column + P at surface

$$P(z) \simeq P_{atm} + \rho g (z \text{ below surface}) \simeq 1\ atm + z(m)/10\ atm$$

$$= \int_{z_{atm}}^{z} \rho g\, dz$$

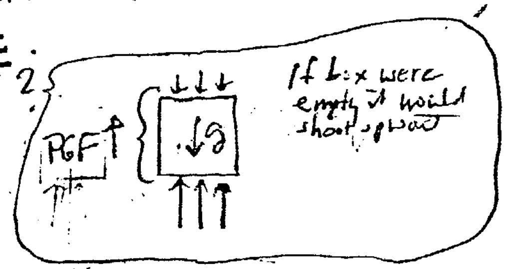
$1\ atm = 14.7\ lb/in^2 \simeq 10$ m of water $= 1000$ cm $= 1000$ mb $= 10^6\ dy/cm^2$

Remember 1 cm of water ~ 1 mb ~ $10^{-3}$ atm

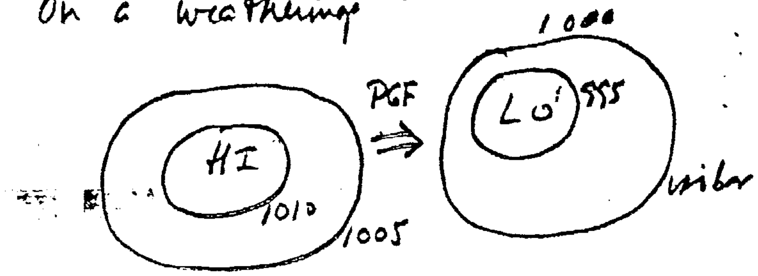


6. Pressure Gradient Force  Although the atmosphere exerts 14.7 lb/in² on every square inch of us, we don't fall over. So net force due to pressure isn't just pressure. Net pressure force due to pressure difference — Pressure Gradient Force.

WHY DOESN'T gravity move fluid downward?

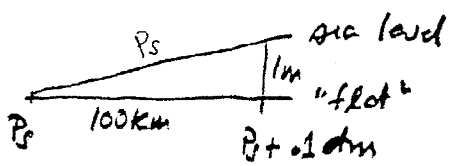



On a weather map




Meteorologists measure PGF directly with barometers. They put lots of barometers at sea level, contour isobars, estimate horizontal pressure gradients. PGF/mass $= -\frac{1}{\rho}\frac{dP}{dx}$  units $\frac{cm^3}{gm}\left(\frac{gm\,cm}{sec^2}\frac{1}{cm^2}\right)\frac{1}{cm} = \frac{cm}{sec^2}$

In the ocean, horizontal pressure gradients are important.
Typical (large) — — — across the Gulf stream
corresponds to 1 meter height difference across 100 km GS.

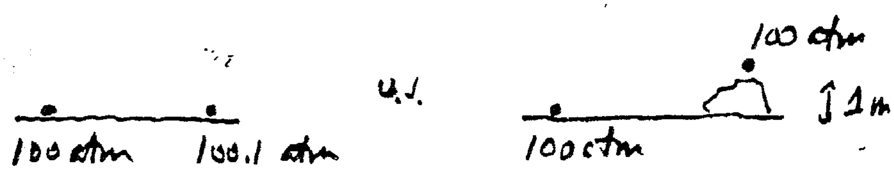



Horiz. accel = Horiz. PGF/mass

$= g\,\Delta h/\Delta x$

$= 10^3\,10^2/(10^2 \cdot 10^5) = 10^{-2}\ cm/sec^2$

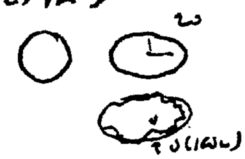
7. Can oceanographers measure horizontal pressure gradients? NO. Why not, meteorologists use barometers?
Meteorologists can do it because air is so light that it doesn't matter if one barometer is on floor & another — in next room — on floor. But water is so dense that putting one barometer on a 10 cm high rock by accident ~~will~~ may reverse estimated HPG




This raises a fundamental #: How do we find a flat surface / plane in ocean?

8. What *is* flat? We call flat surface GEOID.
If it were solid, frictionless, nothing would roll downhill on it.

It's not a sphere, because earth's rotation flattens earth
But it's not a flattened sphere either because earth's rock... are not all same density.



Simplest description that is correct: it's a spheroid whose equatorial radius is about 20 km greater than the polar radius + deviations of a few hundred meters over km to basin scales.

dev from sphere >> dev from spheroid >> dev from geoid.

[Show slide] [Seasat GEOID] ocean

Would be sea surface if ocean didn't move. Unfortunately that difference is crucial

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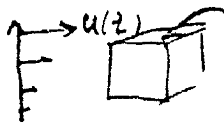
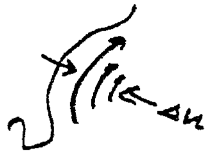
9. Friction Force. Due to velocity gradient. Consider parcel in shear
ie not $\nu\, du/dz$ but gradient



$$\frac{x\text{-Force}}{\text{area}} = \rho\nu \frac{du}{dz}\Big|_{z+\Delta z} - \rho\nu\frac{du}{dz}\Big|_{z} = \rho\nu \frac{d^2u}{dz^2}\Delta z$$

$$x\text{-accel} = \frac{x\text{-Force}}{\text{area}} \times \frac{\text{area}}{\text{mass}} = \rho\nu\frac{d^2u}{dz^2}\Delta z \frac{\Delta x \Delta y}{\rho \Delta x \Delta y \Delta z} = \boxed{\nu \frac{d^2u}{dz^2}}$$

In Gulf stream, of width $W$, $d^2u/dz^2 \sim \Delta u/W^2$



$$x\text{-accel} = \nu \Delta u/W^2 = \frac{(.01)(100)}{(100 \cdot 10^5)^2} = 10^{-14}\ \text{cm/sec}^2$$
$<<$ PGF

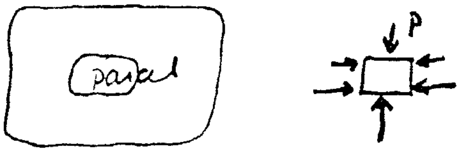
Even if $\nu \to$ Kelly $= 10^8$, $\nu\Delta u/W^2 = \frac{10^8 10^2}{10^{14}} = 10^{-4}$ cm/sec$^2$ $<<$ PGF

So, in horizontal force balance $|PGF| >> |\text{Friction}|$
What balances PGF (something must if flow is steady)?

Another way to see that another force is needed - pf

10. Now stop cataloging forces + consider some situations

10a Fluid at rest in gravity field. PGF's push up, gravity pulls down

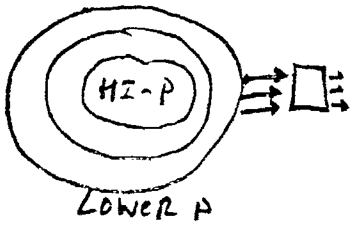



⇒ PGF ↑ gravity ↓ PGF + gravity = 0

No vertical acceleration; hydrostatic.
If ~~[illegible]~~ pressure is constant at same level (such as flat free surface) then no horiz acceleration either

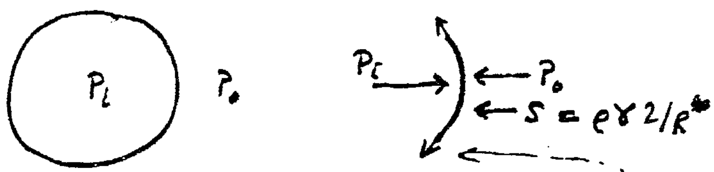
What happens if the parcel is a bubble?
– – – – – – – crown of gold?

10b. Explosion. Just after reactants combust.




Radial acceleration = PGF/mass
Parcel is accelerated outwards.
Unsteady flow

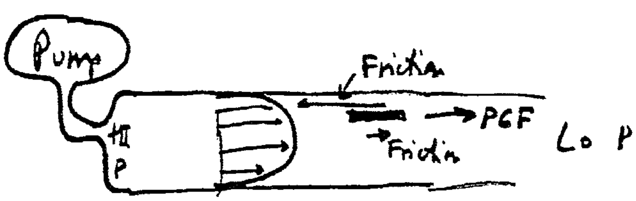
10c. Bubble in air. $P_{inside} > P_{outside}$




$$P_o/\rho + \frac{2\gamma}{R^*} = P_i/\rho$$

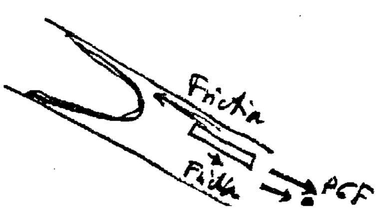
surface acts like a string, "tension" makes $P_i > P_o$

10d. Pipe. Pump forces fluid through

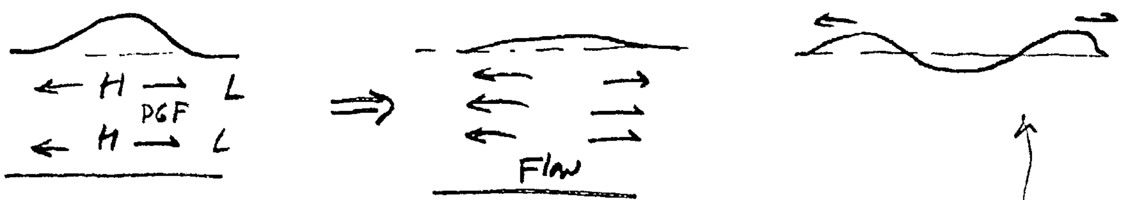



steady

10e Tilted Pipe



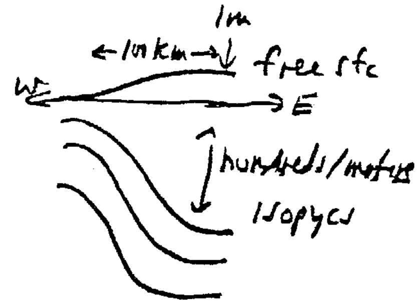
10f Collapse of 'heaped up' free surface




Water drains out from under initial mound, because PGF pushes it horizontally outward. can over-drain ... wave

In stratified fluid, same thing happens if isopycnals are disturbed

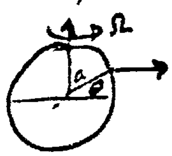

YET IN OCEAN ACROSS GULF STREAM:
it doesn't collapse!!!




11. Then there are more forces. Centrifugal & Coriolis.
We must acknowledge them because we want to measure motion relative to rotating earth – an accelerated reference just like the floor of an elevator

Imagine being on merry go round. Sit still on your horse.
You feel thrown radially outward. Centrifugal force, $accel = \Omega^2 r$ $\quad \Omega = \frac{2\pi}{T}$

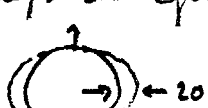
Apply to earth




$\Omega^2 a \cos\theta$

$a = 6000$ km
$\Omega = 2\pi/86400$ sec
$\Omega^2 a = 3.17$ cm/sec$^2$

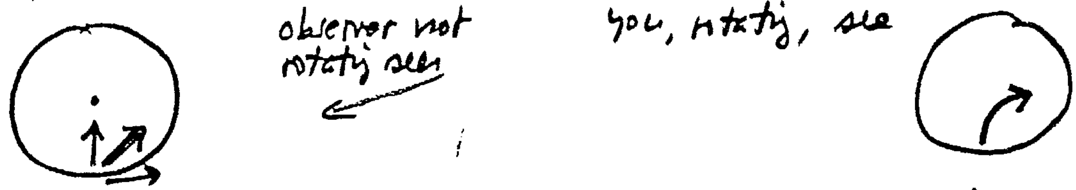
Tend to weigh .3% less at equator than pole.
If solid earth were true sphere, motionless ocean would be 20 km deeper at equator than at pole. WE DONT SEE ANYTHING LIKE THIS. CONCLUDE:




SOLID EARTH ITSELF IS SPHEROIDAL, $r_{eq} \simeq r_{pole} + 20$ km. Geoid ± O(±100m)

Thus: horizontal g + horizontal centrifugal force = 0. So we can neglect centrifugal force because earth is spheroidal

__Coriolis Force__ return to merry go round. Sit at rim, try to throw pebble towards center. When no rotation — hit center. When rotation viewed from above is CCW (like Earth from polaris), hit to right of center.




You'll think: I aimed at center, but after I'd thrown pebble some force deflected it to right.

That is Coriolis force. Properties

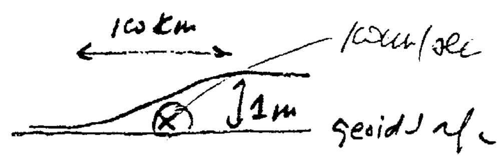
1. Acts on moving objects
2. Proportional to their speed
3. 90° to right of motion in NH
   – – left – – – SH
4. Biggest at poles, → 0 on equator

$|accel| = 2\Omega \sin\theta \times |speed|$

Try it out at children's playground / Del Mar fair!!!

12a { Flow situation with Coriolis.

Remember Gulf Stream

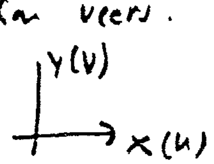



$PGFA = g \frac{\Delta h}{\Delta x} = \frac{10^3 10^2}{100\ 10^5} = 10^{-2}$ cm/sec² to west

$CFA = 2\Omega(\sin 45°)v = 2 \frac{2\pi}{86400} .707\ 100 = 10^{-2}$ to east.

They balance!!! This will be important

12b. Coriolis force alone. Start motion impulsively — thereafter if Coriolis is only force, acceleration is always to right of motion. Motion veers.




$u_t = (2\Omega \sin\theta) v$

$v_t = -(2\Omega \sin\theta) u$ ⇒


$T = \frac{2\pi}{2\Omega \sin\theta} = \frac{12\ hr}{\sin\theta}$

$2\pi r = V_0 T$  $r = \frac{V_0 T}{2\pi}$

$V_0 =$ 10's cm/sec

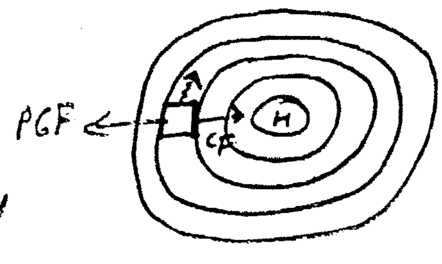
$r =$ km.

See motion like this very frequently in moored current meter records — at local inertial period $12h/\sin\theta$ ± few %.

Pole 12 hr
30 24
Eq ∞

related to Foucault Pendulum

## 12c Geostrophic / Weathermap flow




On weathermaps around H

(i) PGF is outward

(ii) If CF = PGF, CF is inward

(iii) ∴ flow is CW around H

Meteorologists measure P, say PGF/mass = $(2\Omega \sin\theta)$ speed

ie interpret P maps for winds. This is why oceanographers want to measure P.

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But oceanographers measure fluid density $\rho(x,y,z)$. Then can say $P_{\text{one depth}} = P_{\text{another}}$ + weight of column of known density

$$\frac{PGF_{\text{one depth}} - PGF_{\text{another}}}{\text{mass}} = (2\Omega \sin\theta)(\text{speed}_{\text{one depth}} - \text{speed}_{\text{another}})$$

ie oceanographers interpret relative P maps to find relative currents. They get velocity differences between levels in the ocean.

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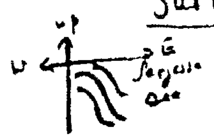
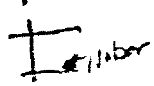
Either draw such maps, say 0/1000 m, or look at sections.

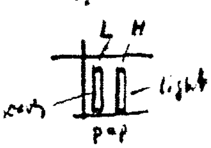
Gulf Stream Section




(i) observe T, S, get density. Isopycnals plunge hundreds of meters from on- offshore Gulf Stream.



(ii) suppose no flow at some great depth, ie. there P = con

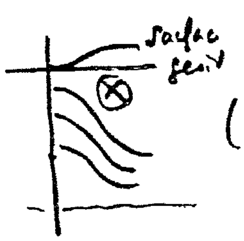



(iii) So upward from that level is cold dense near shore column & warm light Sargasso sea column; p falls faster in denser column

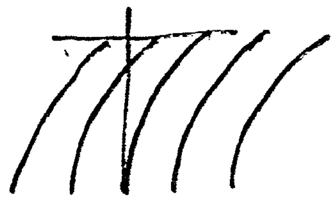
(iv) So at higher levels, Sargasso Sea p > nearshore p

(v) if geostrophic, CF is eastward in these levels.
So flow is northward

(vi) ~~[illegible]~~
Since p gradient persists to very surface, surface itself must tilt upward relative to geoid from shore - seaward



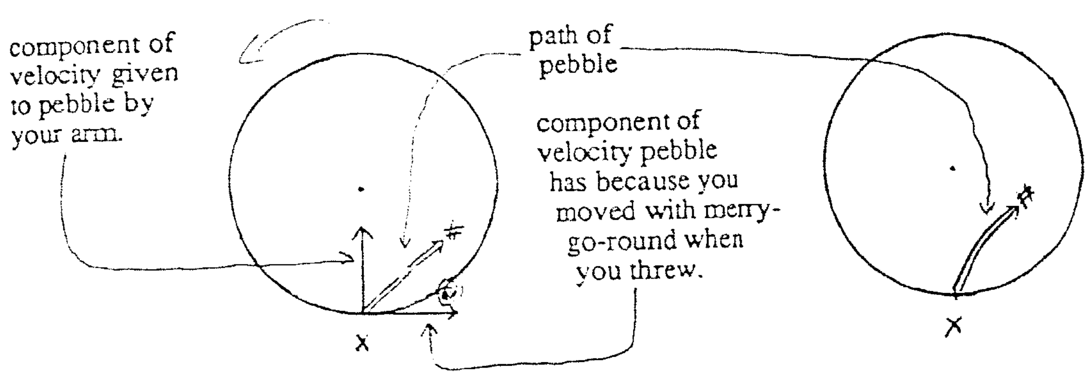

(vii) Flow profile could be any of ↙ if all we know is density. Further knowledge of flow at one depth / surface etc, in principle fixes profile.



CORIOLIS FORCE. You may experience the Coriolis force directly at a playground where there is a merry-go-round. Stand or sit at the perimeter of the merry-go round when it is rotating counter clockwise as viewed from above (the sense in which the earth rotates when viewed from the pole star). Attempt to throw a pebble directly towards the center of the merry-go-round. You will see the pebble veer off towards your right, it will not hit the center of the merry-go-round. The force that causes this veering is the Coriolis force. The reason it exists is explained in the following diagrams:

Rotating merry-go-round viewed from above by a fixed (non rotating) observer.

Rotating merry-go-round viewed from above by an observer rotating with merry-go-round.




Your starting location is x in both diagrams. The fixed observer sees you move to @ while he sees the pebble go to #. To the moving observer you don't change position but the pebble moves to #. In both diagrams pebble starts at x and is later at #, not at center of merry-go-round.To the observer rotating with merry-go-round, this is because Coriolis force deflected pebble. To the non rotating observer it is because pebble didn't start out moving towards center; it started with one component of velocity towards center and another along perimeter. This experiment correctly shows that (a) Coriolis force is exerted only on moving objects (if you don't throw the pebble it doesn't veer), that (b) Coriolis force on a moving object is to the right of the horizontal direction of motion of the object in the northern hemisphere. Also true but not clear from this experiment are that (a) Coriolis force is exactly 90 degrees to right of motion in northern hemisphere, (b) Coriolis force is exactly 90 degrees to left of horizontal direction of motion in southern hemisphere (you can see the change in direction by redrawing the diagrams with a clockwise rotating merry-go-round), that (c) Coriolis force is greatest at poles, vanishes at equator, and that (d) Coriolis force doubles, triples ... if object's speed doubles, triples ... .

WEATHERMAP i.e. GEOSTROPHIC FLOW IN THE ATMOSPHERE. In the atmosphere, air pressure at sealevel varies from place to place. The average pressure is one atmosphere i.e. 1000 millibars (mb). In a severe hurricane, sea level pressure might drop as low as 960 mb, but in most high and low pressure systems it ranges between about 1030 mb and 970 mb. Its range is thus about three percent of its average value. These place-to-place differences in sea level air pressure are ultimately caused by solar heating of the atmosphere and by cooling of the atmosphere when it radiates energy to outer space.

Meteorologists measure sea level pressure with barometers, which effectively give air pressure at their location on a dial. Meteorologists maintain many barometers at many different locations, and every day they use their readings to draw a map of sea level air pressure. Such a map is given below for the north Atlantic ocean and the eastern US. The solid lines labelled in mb are lines along which the pressure has the labelled value, they are called isobars or lines of constant pressure. Isolated regions of high or low pressure are labelled H or L, respectively.

Why do meterorologists draw these maps? It is because these maps tell us how the air moves at sea level, i.e. they tell us sea level winds without our having to measure the winds themselves (it is much easier to install and maintain a barometer than an annemometer). You might think that if there were an isolated region of high pressure on such a map, then the wind would blow horizontally away from that high pressure region towards surrounding low presure regions. If you think this, your intuition has correctly told you that pressure forces on air parcels are directed from high pressure towards low pressure. Such forces are called pressure gradient forces because they are produced by place-to-place pressure differences i.e. pressure gradients.

But air does not move horizontally away from regions of high pressure. Instead, it mainly moves around high pressure regions - in the clockwise sense viewed from above. Why? This happens because in the atmosphere, the pressure gradient force away from the high pressure center is almost exactly balanced by the Coriolis force; the Coriolis force is thus towards the high pressure center. The Coriolis force is however to the right of the direction of motion of the air on which it acts in the northern hemisphere, so that motion itself has to be clockwise around the high pressure center.

Thus for example, at the point marked # on the sea level air pressure map below, the pressure gradient force is towards the west (from high towards low). The Coriolis force has thus to be towards the east (opposite to the pressure gradient force). The Coriolis force is exactly to the right (90 degrees) of the wind, so the wind must be northward.

You can repeat these arguments to convince yourself that, viewed from above, winds blow clockwise around northern hemisphere high pressure centers and around southern hemisphere low pressure centers, but counter clockwise around northern hemisphere low pressure centers and around southern hemisphere high pressure centers.

A common mistake: people sometimes look at the flow around a northern high pressure center in the atmosphere or the ocean and say "the reason the flow curves to the right is because the Coriolis force pushes fluid parcels to the right of their direction of motion." This statement is wrong. You can see that it would give the wrong curvature of flow around an isolated low pressure center. In both cases, the flow pattern is determined by the pressure field; once you have the pressure map you can convert it to flow like a meteorologist. So the real question is, what determines the pressure field? A weathermap doesn't tell us directly (although a forecaster uses it in conjunction with lots of other information to forecast changes in the pressure field).

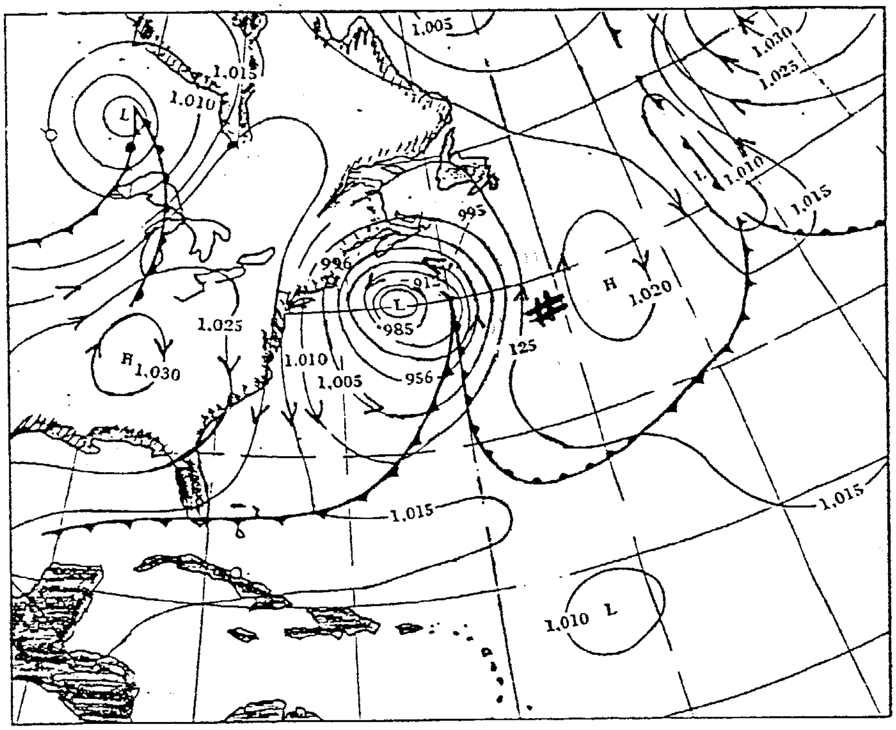



Fig. 9.1 A simplified weather chart (November 20, 1955, 1200 GCT) showing the manner in which the sea-level winds are related to the distribution of pressure. In the Southern Hemisphere the winds would blow with low pressure *L* on their right, and high pressure *H* on their left. The heavy lines with attached symbols show the position of the polar front.

WEATHERMAP i.e.GEOSTROPHIC FLOW IN THE OCEAN. Meteorologists measure sea level air pressure with a barometer. Air is so light that, for practical purposes, we get the same barometer reading whether we put the barometer in the attic or in the cellar. But water is much heavier (technically, much more dense) than air. If we put a water-proof barometer on the ocean floor at exactly 4000 meters depth below the sea surface, it will read about 401 atmospheres (one from the atmosphere and about 400 from the 4000 meters of water, roughly one atmosphere for every 10 meters of water). It will thus read about 401,000 mb (401 atm x 1000).

To make a "weather map" of pressure at the sea floor we would have to put lots of barometers on the sea floor and record their readings, then draw the map. If we could do this, we could then draw sea floor ocean currents on our map just as meterologists draw sea level winds on a sea level air pressure map. We would like to be able to do this, because it is very expensive to put current meters on the sea floor and recover them, and they are prone to mechanical failure. But imagine putting out many barometers in a region where the sea floor is 4000 meters below the sea surface and is perfectly flat, except for one rock a meter high. If one of our barometers landed on the rock instead of beside it, it would read 100 mb too low, 400,900 mb instead of 401,000 mb. If we didn't know it had landed on the rock, we might mistakenly believe that there was an intense (100 mb) low pressure center where the rock is. Our map of ocean floor currents would be all wrong there.

This is <u>the</u> problem with measuring pressures in the ocean and using them to get currents. We can make waterproof barometers with plenty of accuracy to make the measurements we need, but we don't know precisely enough where to put the barometers to get accurate horizontal pressure differences. You might think that we should hang the barometers at a precisely measured distance from the actual sea surface. We probably could do this with the requisite precision, even allowing for waves, etc., but some thought will show you that this doesn't solve the underlying problem, which is that we don't know the horizontal direction at the barometers because we don't know the horizontal direction at the surface (it is not just the average sea surface!). We will return a the more precise statement of this difficulty in the next section. To do the things described in the rest of this section, we don't <u>have</u> to know the horizontal direction very well.

Even though we thus can't determine horizontal ocean pressure differences directly, we can easily measure horizontal differences between the weights of different water colums of the same length in the ocean. The pressure difference between the top and the bottom of a column of seawater whose area is one square centimeter is just the weight of that column; we can and frequently do get that weight easily just by measuring the temperature and salinity of the water at various heights in the column and then using standard formulae to calculate the weight of the column (more technically, we use temperature and salinity to calculate the density of the water and from that we get its weight). So once we know the horizontal distribution of the weight of seawater columns between any two levels, we can draw a map of the <u>pressure difference</u> between those levels and then follow the meteorologists' reasoning to get a map of the <u>difference in horizontal flows</u> between the two levels. We can do this without having to know to within a few centimeters where the actual tops and bottoms of the colums are, all we need to know are their weights.

That is what has been done in the example shown below. Measurements of temperature and salinity at between perhaps five to as many as perhaps fifteen different heights in the water column from the surface to 1000 meters have been made at each one of the many locations shown by dots. The weight of the column at each dot has been calculated, and isolines of the weight difference between the surface and 1000 meters have been drawn. Thus for example surface-to-1000 m colums lying below the line labelled <u>1.5</u> weigh more per unit area than those lying along the line labelled 1.4; the difference in weight is the weight per unit area of a column 0.1 meters high. You may interpret this map to get the flow just as you do a regular weathermap, except that now the flow around, for example, the high pressure difference center (marked H) just east of Japan, is the surface flow relative to whatever flow may exist at 1000 meters. So a map constructed this way doesn't give us the flow itself at some level, instead it gives us the flow at one level relative to the flow at another, it gives us the difference between the flows at the two levels. We learn the flow difference between levels but not the flow at either individual level.

When you look at the surface-1000 meter map below you will however recognize most of the features of the Pacific surface circulation that you can see on the accompanying current map derived from ship drift data. This means that the difference between the surface flow and the flow at 1000 meters is dominated by the surface flow itself, i.e. that the surface flow is much more intense than the flow at 1000 meters. This is generally true in the world ocean.

You might (should!) be surprised that two such different data sets (temperatures and salinities on on hand, and ships drifts on the other) give such similar surface circulations. On both maps you see the subtropical gyres in northern and southern hemisphere, you see the Alaska subpolar gyre, you see the California and Peru currents, and you see the circumpolar current as well. The Kuroshio is also visible but hard to see on the surface-1000 m map simply because it is so narrow that, on the map scale, it is hard to distinguish from the coast of Japan itself. The high pressure center marked H has the the Kuroshio on its western side; but the isobars are so crowded together that you can't distinguish them west of the H on the map.

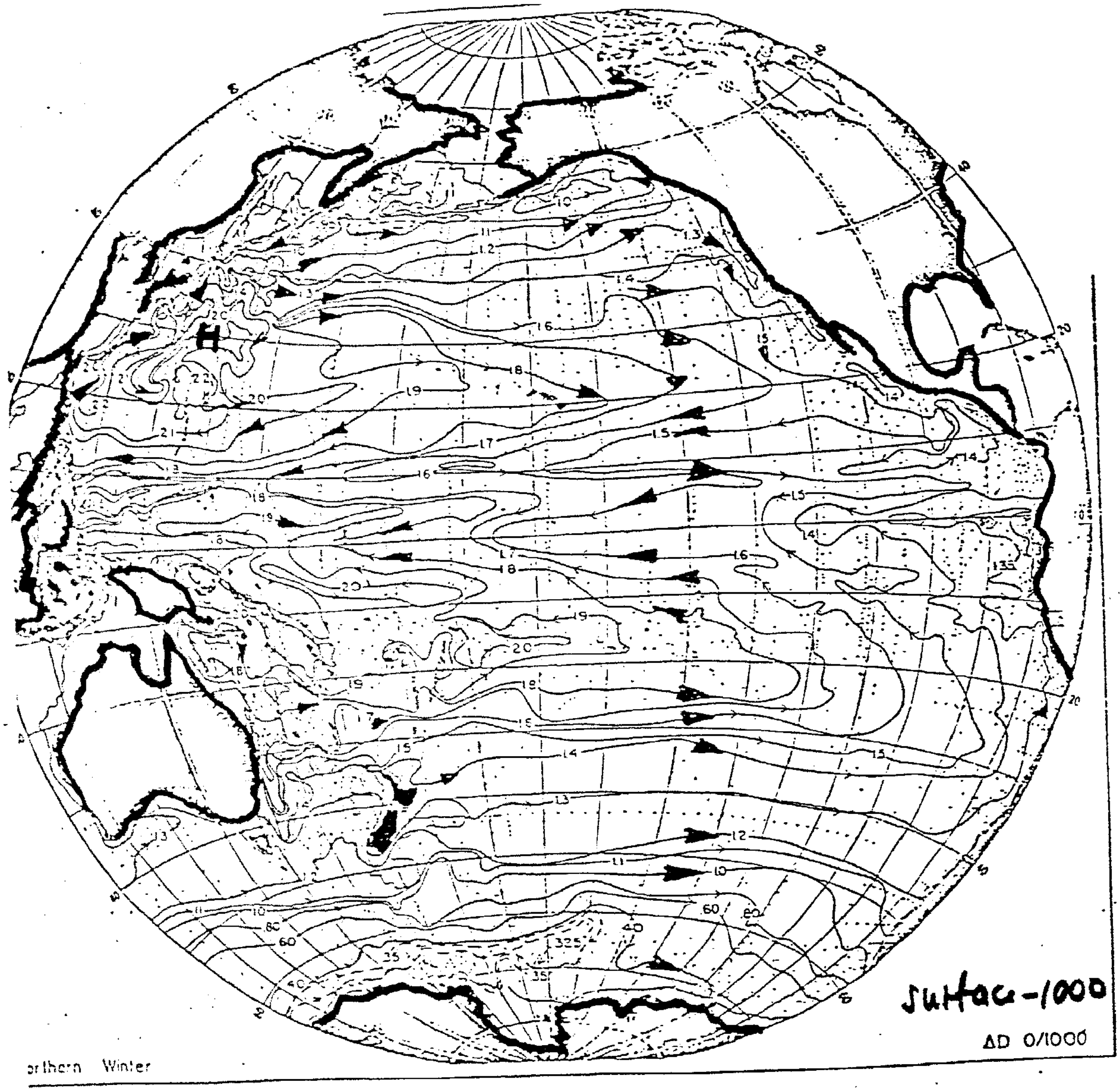
The two maps are similar in their general features but very different in details. This is not surprising when you realize that they are two very different smoothed approximate views of the circulation; the temperatures and salinities were taken at many different times at stations located very irregularly whereas the ships drifts were taken over many years and then somehow averaged to estimate the current (they probably also reflect windage on the reporting ships).

We've noted that on the surface-1000 meter map isobars are labeled in meters. You can think of these isobars in terms of pressure of you convert them to mb by multiplying by 100. If you do this you will see that horizontal pressure differences across the near-surface ocean correspond at most to about one meter or 100 mb, i.e. about a tenth of an atmosphere. These isobars are also lines along which the sea surface is everywhere the same height above the exactly horizontal surface which we wanted to find earlier in this section but were unable to define. It will be defined in the next section, but for now, notice that sea level varies relative to it by about a meter or a little less from California to the H and then fall by about a meter from the H to Japan (you can't see this last on the surface-1000 map because the isobars are too crowded together, but it would be visible on an expanded presentation).

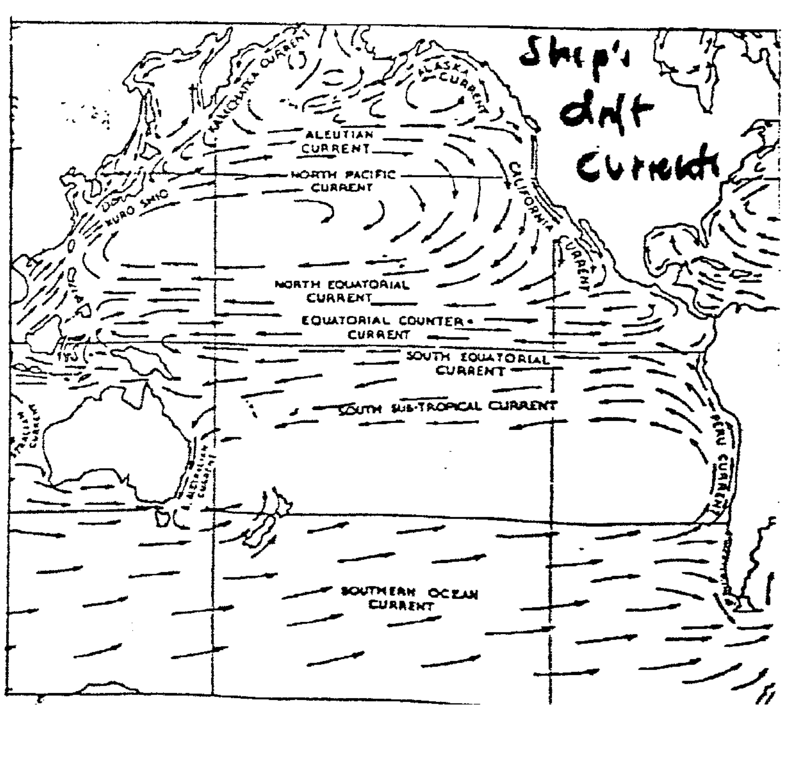
The flows shown on these two maps are near surface flows. They are driven by ocean surface winds. This may seem to contradict the idea that in ocean currents, pressure gradient forces are balanced by Coriolis forces - why are't wind stresses included if the winds drive the currents? The answer is that the geostrophic balance of pressure gradient forces and Coriolis forces holds throughout most of the water column, but very close to the ocean's surface that balance is disturbed by frictional wind-induced stresses. The region in which this occurs is called the Ekman layer (although what happens there is now recognized to be much more complicated than in Ekman's simple picture).

The Ekman layer is only a few tens of meters thick. Wind-induced stresses are large at its top (the sea surface) and virtually vanish below it. Although the flow profile from top to bottom of the Ekman layer is complex, the total wind stress driven transport of water from top to bottom of the Ekman layer is simple - that transport is 90 degrees to the right of the wind in the northern hemisphere (90 degrees to the left in the southern hemisphere). Thus under northern hemisphere mid-latitude westerlies (winds from the west) the Ekman transport is equatorward while under northern hemisphere tropical trades (winds from the east) it is poleward. The prevailing pattern of tropical trades and mid latitude westerlies thus drives Ekman layer waters together between the trades and the westerlies. This water has to go somewhere, it sinks down out of the Ekman layer (at speeds of several tens of cm per day) in the centers of the major subtropical gyres. It is this sinking that actually causes the water below the Ekman layers to move; that motion is geostrophic (pressure gradients and Coriolis force balance) because the wind stress is only appreciable within the Ekman layer.

So the wind driven circulation is geostrophic below the Ekman layer. The wind driven circulation at the ocean's very surface consists of this geostrophic part plus the Ekman layer flow itself. The Ekman layer flow is however not strong enough to make the pattern of very-surface and sub-Ekman-layer flow unrecognizably different. The surface-1000 m map below shows the sub-Ekman-layer flow, the ships drift map approximates the very-surface flow.



ΔD 0/1000
orthern Winter

Ship's drift currents
KAMCHATKA CURRENT
ALASKA CURRENT
ALEUTIAN CURRENT
NORTH PACIFIC CURRENT
KURO SHIO
CALIFORNIA CURRENT
NORTH EQUATORIAL CURRENT
EQUATORIAL COUNTER-CURRENT
SOUTH EQUATORIAL CURRENT
SOUTH SUB-TROPICAL CURRENT
PERU CURRENT
SOUTHERN OCEAN CURRENT

INTERPRETING A HYDROGRAPHIC SECTION FOR THE SENSE OF GEOSTROPHIC FLOW. We've just seen that hydrographic (temperature and salinity) data can be used to construct geographical maps of the flow at one level relative to that at another. It is however more common to display that data on a vertical hydrographic section. Such a section, this one across the Gulf stream, is shown at left below. Here temperatures have been measured at a number of depths within the columns whose numbered labels appear at the top of the section. Water is warm near the surface (top of the figure) and coldest near the sea floor (bottom of the figure). Isotherms (solid but curved lines along which the temperature has the labelled value) dip downward through and under the Gulf stream so that they are several hundred meters lower in the Sargasso sea (east of the Gulf stream, to the right on the figure) than over the US continental shelf (west of the Gulf stream, towards the left edge of the figure).

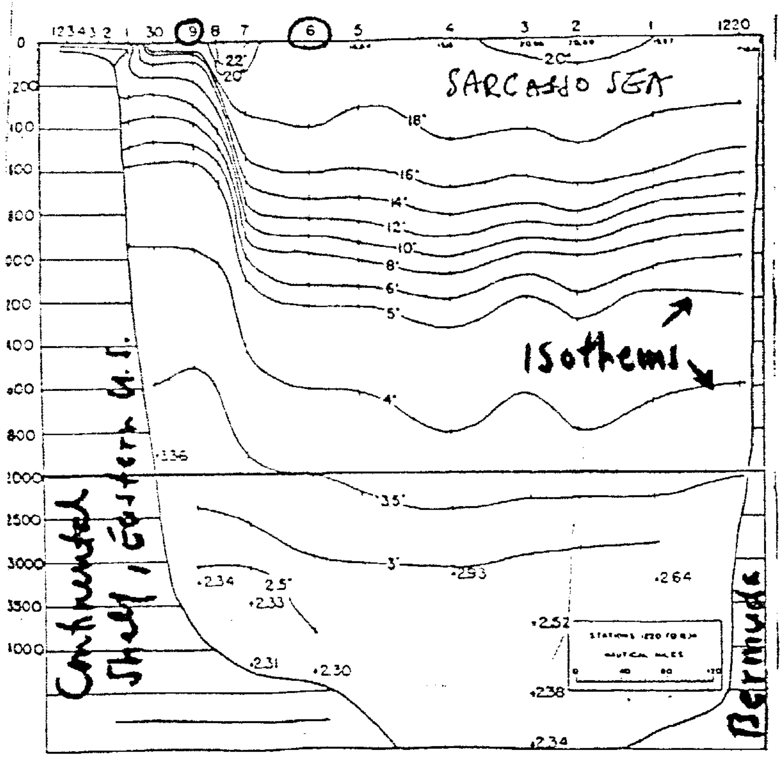
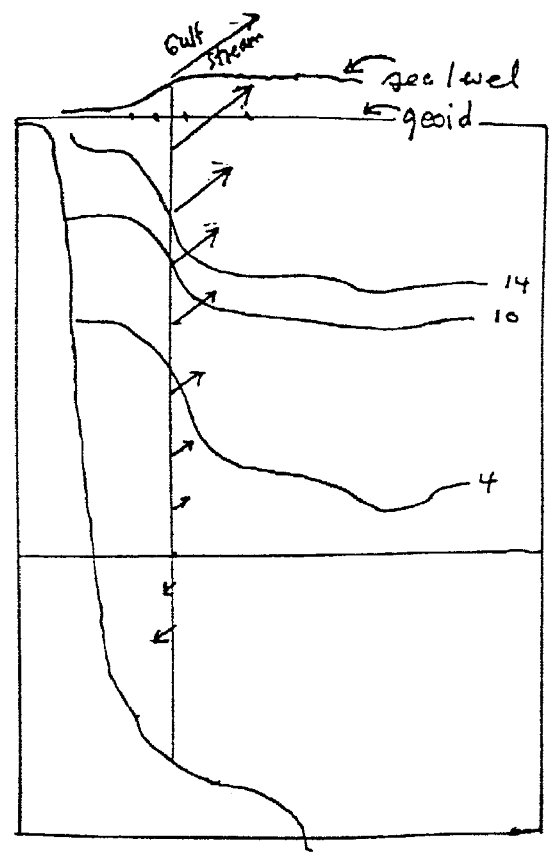



Fig. 20. Temperature section across the Gulf Stream, Chesapeake Bay to Bermuda, April 17–23, 1932, according to Iselin (1936, fig. 5).




What does such a section tell us about the geostrophic flow? The explanation goes this way. Suppose that at some fairly great depth, perhaps about 2000 m on this section, the flow is very slow (the fact that in the previous section the surface-1000 m flow was mainly surface flow encourages us to make such an assumption). Horizontal pressure differences at this depth will therefore be very small. For the sake of argument momentarily suppose the pressure is the same at all stations at 2000 m on the section above. Now imagine going upward from 2000 m through both column 6 in the Sargasso sea and column 9 over the shelf. As you go up, the water pressure in both columns will decrease but the pressure in column 6 will decrease more slowly than the pressure in column 9 because the water of column 6 is warmer and therefore lighter than the water of column 9. So when you have gone upward equal distances in each column from the 2000 m level to just a few meters below the sea surface, you will find a higher pressure in column 6 than in column 9. On this section it will be about 100 mb higher.

Just below the surface then, the pressure gradient force on water parcels between columns 9 and 6 is from high to low pressures i.e. it is westward, towards the left on the figure. As in the atmosphere, it will be balanced by the Coriolis force. The Coriolis force on these water parcels is therefore eastward, towards the right on the figure. The Coriolis force is to the right of the direction of flow in the northern hemisphere; the flow of these near surface parcels of water between colums 9 and 6 must therefore be northward, into the plane of the figure. That flow is the Gulf Stream.

The foregoing argument shows you that the horizontal pressure gradient force is greatest near the surface and decreases with depth. We assumed it decreases to zero at 2000 meters; if this assumption is correct it may reverse direction below 2000 m. Correspondingly, the flow is into the plane of the figure i.e. northward most strongly at the surface, decreases in strength to no flow at 2000 m, and may reverse i.e. be southward at even greater depth. The part of this flow between stations 7 and 8 is sketched in perspective in the right hand panel of the figure above.

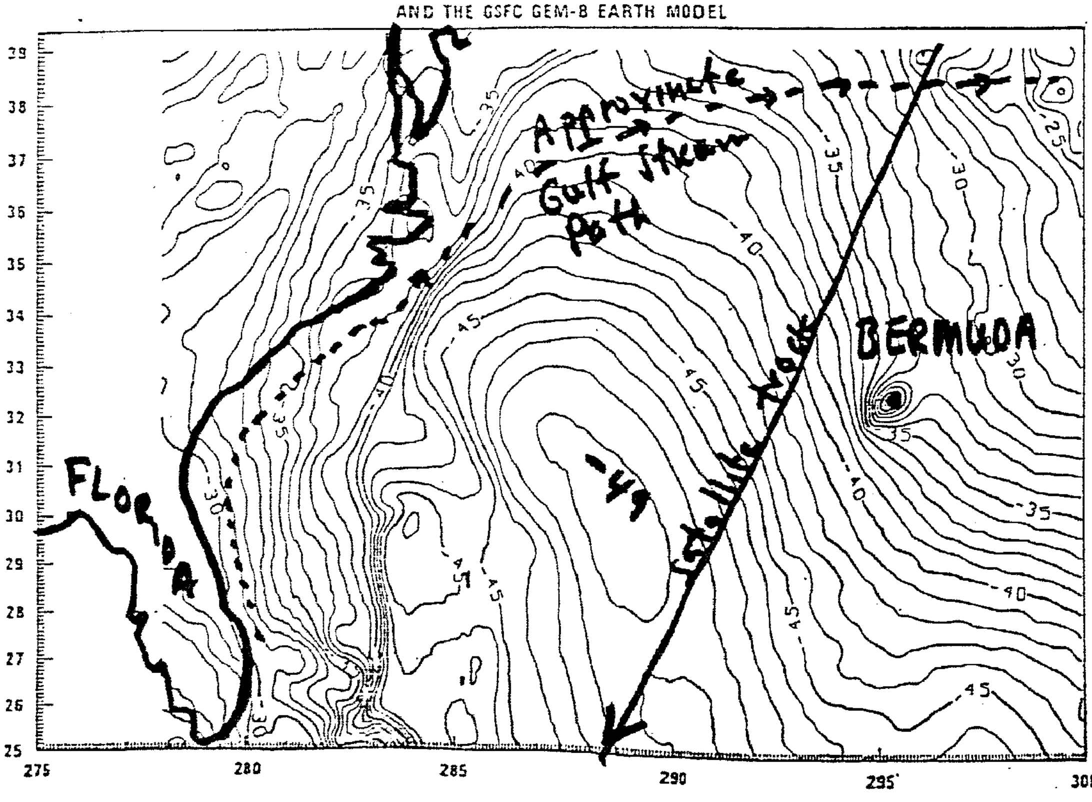
The argument just given is really the same one given in the last section, where maps of the flow at one level relative to that at another were drawn. Here, for the sake of argument, we started by saying the flow vanished at 2000 m. It doesn't exactly, and need not do so even approximately. We can however completely correct our statements about the flow in or out of the plane of the figure by saying they are about the flow relative to the flow at 2000 m.

**You should memorize the form of the Gulf Stream section shown here, with its isotherms or isopycnals sloping downwards from left to right, and remember that in the northern hemisphere this slope of isopycnals in a water column means that water at a given depth in the column flows into the plane of the figure relative to deeper water in the same column.** When you see a new northern hemisphere section, you can apply this rule by rote to get the sense of relative geostrophic flow between levels. If you try to reason it out from first principles each time, you'll probably be wrong about half the time (at least initially). In the southern hemisphere, a section showing this sense of isopycnal slopes means that water at a given depth flows out of the plane of the figure relative to deeper water.

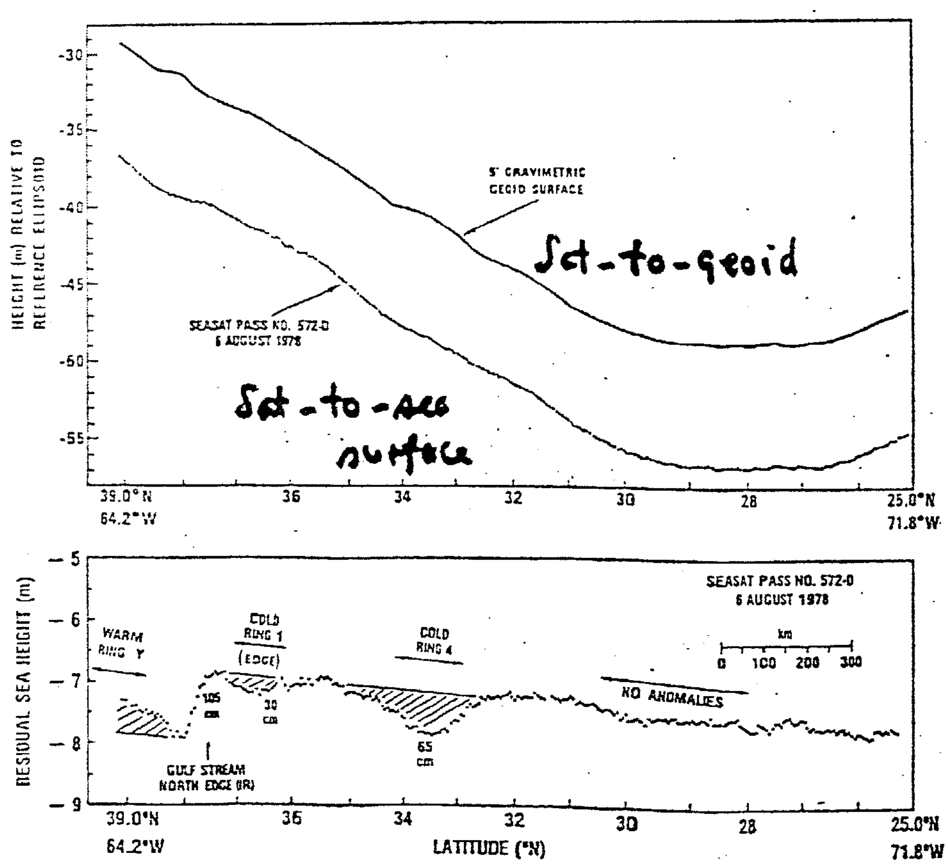
In the Gulf stream, and in most other locations, the deep flow is substantially gentler than the surface flow. Arguments like those above thus predict intense near-surface flows with correspondingly big horizontal pressure differences just below the sea surface. How can such pressure differences arise? They arise because the actual surface of the ocean is a little higher over the high pressure regions than over the low pressure ones. Thus on the Gulf stream section above, the actual surface of the sea (with surface waves and tides averaged out) at column 6 is about a meter higher than the surface of the sea at column 9. The departure of the sea surface from the local horizontal is sketched on the right hand panel of the figure above.But this situation is still hard to understand, because we haven't said with respect to what the sea surface is higher or lower, we haven't said what we mean by horizontal.

WHAT IS HORIZONTAL THEN? Your intuitive idea of horizontal is this; if a frictionless plane were horizontal and we released a ball on that plane, the ball wouldn't begin to slide or roll away in any direction. But if the plane were not horizontal, the ball would be pulled downhill by gravity and would roll away. The technical name for a horizontal surface very close to the surface of the earth is the geoid. The geoid is the shape that the sea surface would have if the ocean didn't move at all, the ocean surface would then everywhere be exactly horizontal. You might think the geoid would be a perfect sphere, but it isn't because the earth rotates and the resulting centrifugal force makes the earth and the ocean bulge out by about 20 km near the equator; the earth's equatorial radius is about 20 km greater than its polar radius (the gravitational attraction of the extra material around the equator is also very important in establishing the ultimate spheroidal shape). Having recognized this you might think the geoid is a spheroid whose equatorial radius is about 20 km greater than its polar radius, but the actual geoid departs from this spheroidal shape by as much as several hundred meters from one ocean basin to another. These departures occur because some parts of the solid earth are more dense than others, so that their gravitational attraction of the water is greater as well. The geoid rises up from the spheroid near islands (their gravity attracts the water) and drops down over trenches (their sides pull the water away from the center). If the geoid were a solid surface, a ball placed on it would't spontaneously roll in any direction, because the force of gravity is always at right angles to the geoid. In this sense, you can think of the geoid as a "flat" or horizontal surface, but it is not a surface which is equidistant from the earth's center or even from some particular spheroidal surface. The following figure is a map of the geoid's elevation relative to the spheroid in the western North Atlantic. You can see that in the center of the mapped region the geoid is many tens of meters closer to the spheroid than it is at the edges of the mapped region. Notice also how Bermuda stands out as a local peak in the geoid.

DETAILED GRAVIMETRIC GEOID
BASED UPON A COMBINATION OF
5' X 5', 15' X 15', 1° X 1° SURFACE GRAVITY DATA
AND THE GSFC GEM-8 EARTH MODEL



If there are near surface ocean currents, there must be near surface horizontal pressure gradients to balance the Coriolis force, as we've seen above. Those near surface pressure gradients occur because the sea surface departs slightly (a meter at most) from being horizontal i.e. from the geoid. Remarkably, the difference between actual sea level (relative to the earth's center) and the geoid (again relative to the earth's center) can be measured from satellites. A satellite carrying an altimeter sends out a radar pulse whose time of flight from satellite to sea surface and back can be measured so precisely that the intervening satellite-to-sea surface distance is known to within a few centimeters. The satellite-to-geoid distance is measured to within a few centimeters by tracking the satellite relative to the earth's center, and simultaneously calculating the geoid under the satellite relative to the earth's center from all the measurements of gravity (both from land and from perturbations of satellite orbits) that have ever been made. The difference between satellite-to-sea surface distance and satellite-to-geoid distance is then the sea level relative to the geoid. An example of how these two distances vary over the Gulf-stream-crossing SEASAT satellite track shown on the previous figure as well as their difference is given in the figure immediately below. You can see not only the Gulf stream itself but also eddies (cold rings) which have spun off the Gulf stream and are drifting in the Sargasso sea.



On the Gulf Stream section shown earlier, just as in the satellite-to-sea surface minus satellite-to-geoid difference shown immediately above, sea level thus rises by about one meter relative to the geoid from column 9 to column 6. If this sea level rise is measured by an altimetric satellite, then we can get ocean currents just below the sea surface by converting the measured sea level rise to pressure at some depth just below the geoid, and then drawing a pressure map which we then interpret just like a weathermap. Notice that we actually measure the near surface pressure itself by using the satellite, so that by this proceedure we get the surface flow itself. Now, at least in theory, we can use the satellite-measured near surface surface pressure and the hydrographically-measured weight of subsurface water columns to get the actual flow at any depth below the surface rather than just the flow difference between levels we got from hydrographic data alone.