SIO 210
11/7/19
Sarah Purkey
Vorticity in Oceanography

Dynamics VI + VII (Lynne)
Reading: Section 7.7.1 through 7.7.4 or Supplement S7.7
(figures are taken from supplementary chapter S7) Section S7.8.1
Chapter S7.7.1 and S7.7.2; S7.8.1, S7.8.3

Vorticity:

- What is vorticity?
- What is planetary vorticity (PV)?
- Conservation of vorticity

Importance of Conservation of Vorticity in oceanography:

- Rossby waves
- Sverdrup balance
- Western boundary currents

Review conservation of vorticity

Potential vorticity (Q):

$$Q = \frac{f + \zeta}{h}$$

f= planetary vorticity
ζ= relative vorticity
h= height

Review conservation of vorticity

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Conservation of potential vorticity:

$$Q = Constant = \frac{f + \zeta}{h}$$

Review conservation of vorticity

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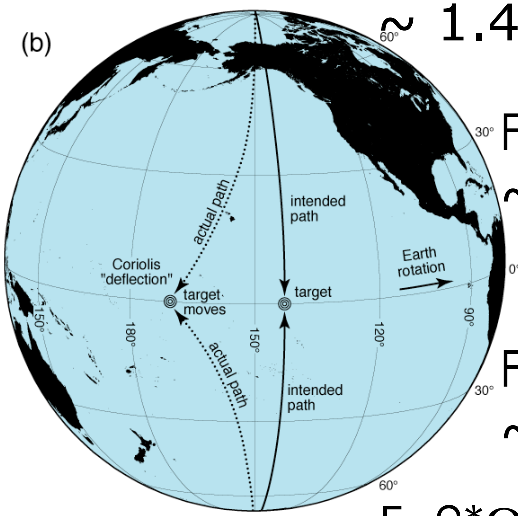
Conservation of potential vorticity:

$$Q = Constant = \frac{f + \zeta}{h}$$

a) How do you increase/decrease f?

Move north: f increases
Move south: f decreases



F=2*Ω sin 90
~ 1.4x 10^{-4}

F=2*Ω sin 30
~ .7 x 10^{-4}

f=0

F=2*Ω sin -30
~ -.7 x 10^{-4}

F=2*Ω sin -90
~ -1.4x 10^{-4}

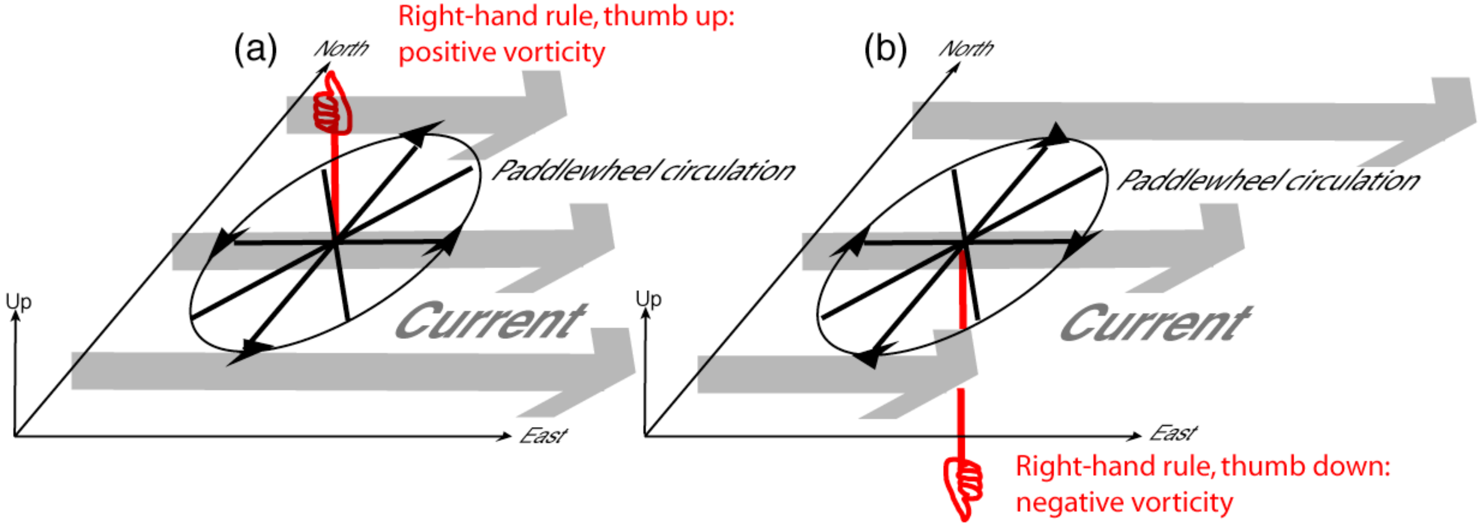
Review conservation of vorticity

f= planetary vorticity
ζ= relative vorticity
h= height

Conservation of potential vorticity:

$$Q = Constant = \frac{f + \zeta}{h}$$

b) How do you increase/decrease ζ ?



Counterclockwise= positive   Clockwise= negative

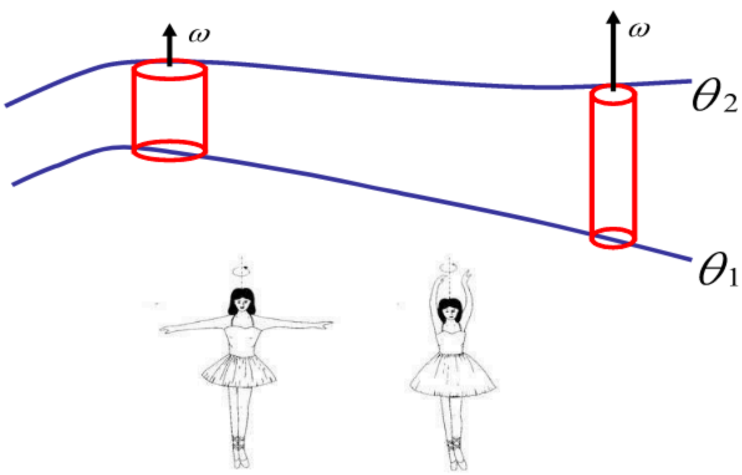
Review conservation of vorticity

f= planetary vorticity
ζ= relative vorticity
h= height

Conservation of potential vorticity:

$$Q = Constant = \frac{f + \zeta}{h}$$

c) How do you increase/decrease h?



Review conservation of vorticity

f= planetary vorticity
ζ= relative vorticity
h= height

Conservation of potential vorticity:

$$Q = Constant = \frac{f + \zeta}{h}$$

a) If h increases and no change in latitude, what happens?

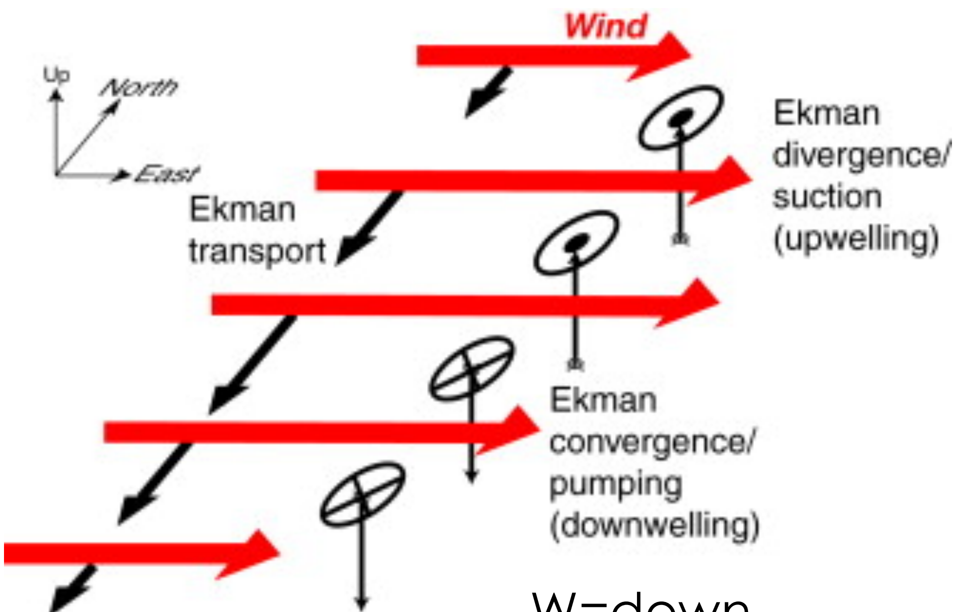
b) A parcel of water moves south but h is not allowed to change, what happens?

c) If a parcel of water move north in northern hemespher and ζ stays the same – what happens?

How does Ekman transport drive underlying circulation?

Northern hemisphere :

$$Q = \frac{f}{h}$$



Stretching isotherm
(H increasing)

Water moves poleward

Squashing isotherm
(H decreases)

Water moves equator-ward)

W=down

W=0

The Math: Sverdrup balance and relation to winds

(1) The vorticity equation for really large-scale flows is:

$$\beta v = f \frac{\partial w}{\partial z}$$

v is the meridional (south-north) flow
w is the vertical velocity.

(2) We vertically integrate this (from bottom of ocean to top) to get the meridional transport V.
We assume that the vertical velocity w at the ocean bottom is 0, and the vertical velocity w at the top is due to Ekman pumping/suction w_{Ek}

$$\beta V = f(w_{Ek} - 0)$$

(3) The Ekman pumping is due to variation (curl) in the wind stress

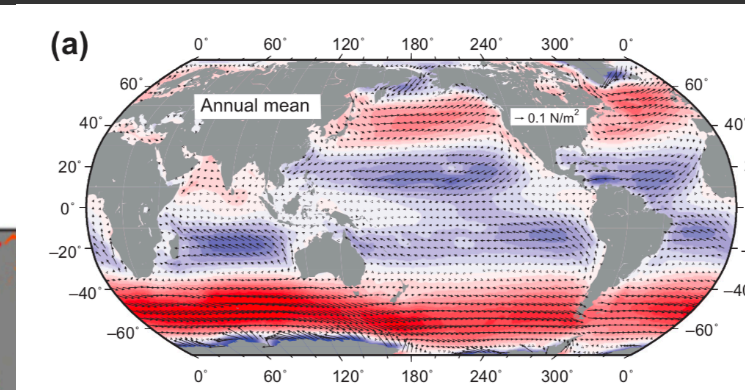
$$w_{Ek} = \left(\frac{\partial(\tau^{(y)}/\rho f)}{\partial x} - \frac{\partial(\tau^{(x)}/\rho f)}{\partial y}\right) = \text{"}curl\text{"}(\boldsymbol{\tau}/\rho f)$$

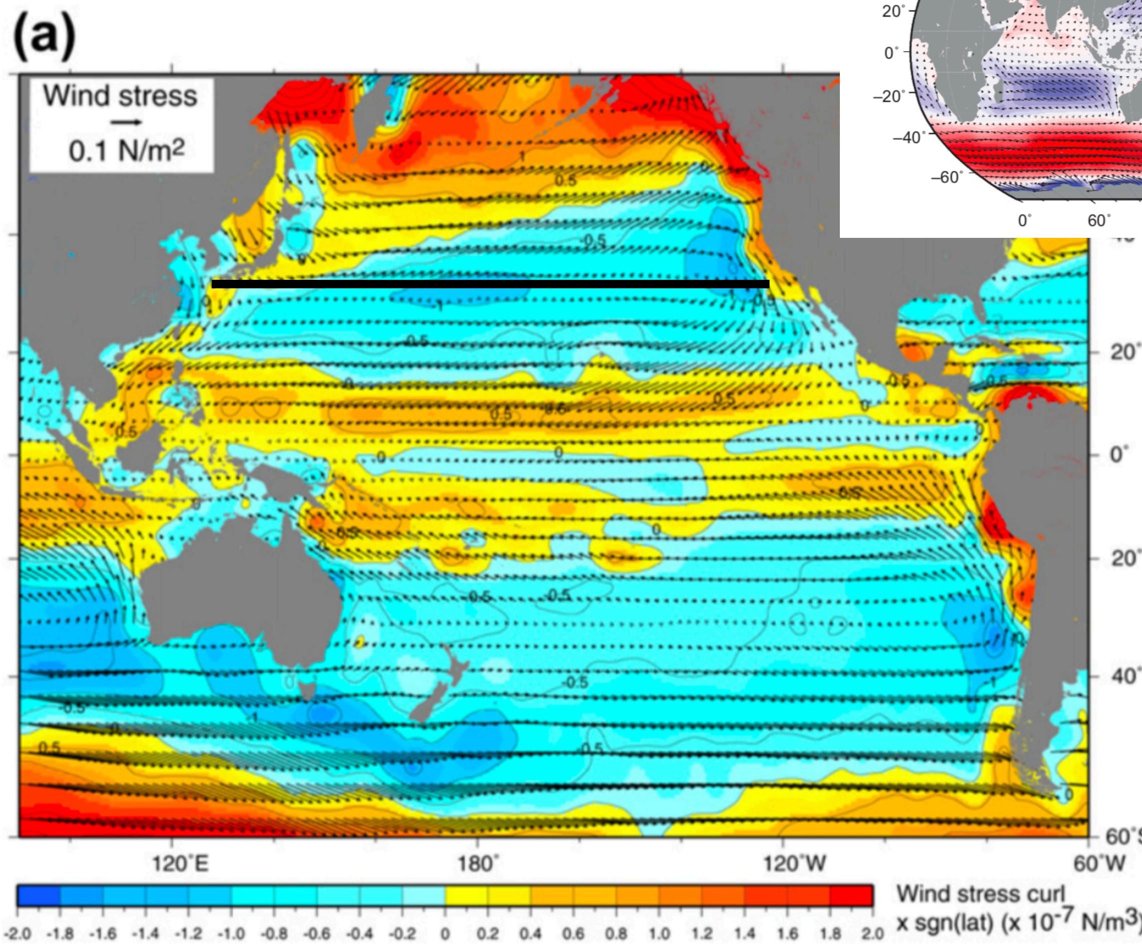
SO therefore $$\boxed{\beta V = f\,\text{"}curl\text{"}(\boldsymbol{\tau}/\rho f)}$$

Sverdrup balance

Data: Ekman upwelling/downwelling

$$\beta V = f\,\text{"}curl\text{"}(\boldsymbol{\tau}/\rho f)$$





Blue regions: Ekman pumping -> equatorward Sverdrup transport

Yellow-red regions: Ekman suction -> poleward Sverdrup transport

Sverdrup Transport (theoretical)

$$\beta V = f\, "curl" (\boldsymbol{\tau} / \rho f)$$

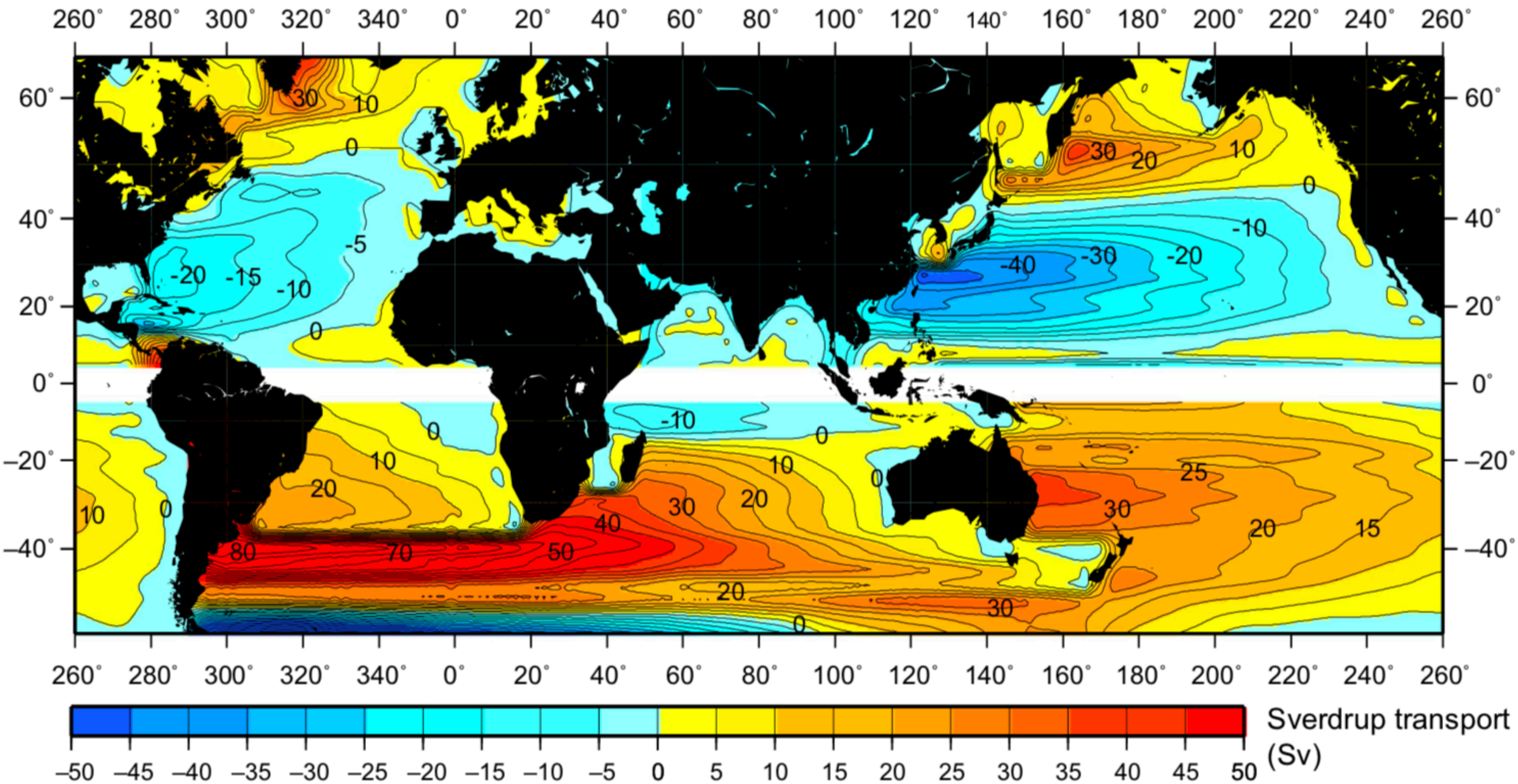
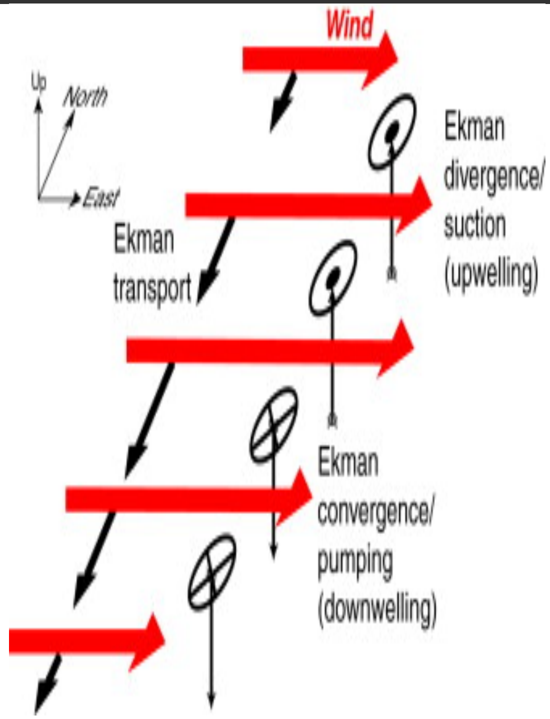
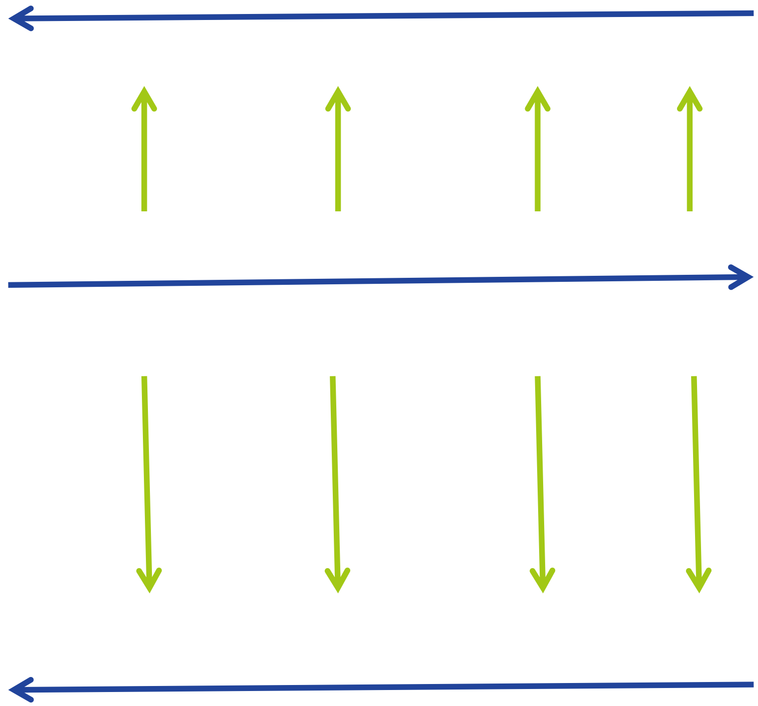


FIGURE 5.17 Sverdrup transport (Sv), where negative is clockwise and positive is counterclockwise circulation. Wind stress data are from the NCEP reanalysis 1968–1996 (Kalnay et al., 1996). The wind stress and wind stress curl used in this Sverdrup transport calculation are shown in the online supplement, Figure S5.10.

Sverdrup Transport





Sverdrup Transport North

Sverdrup Transport south

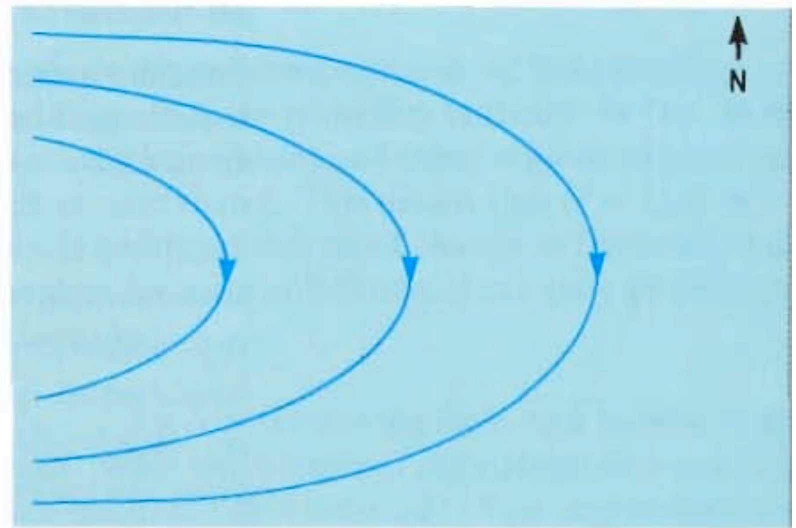
Eq

Why does the southward flow connect to the western boundary and not to the eastern boundary???

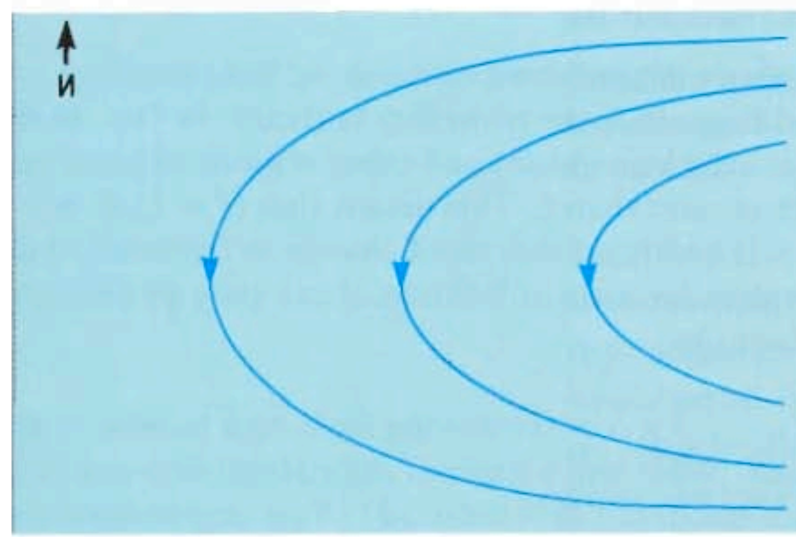
WBC

- If winds create Sverdrup transport, where/how does the water return back to where it started?
- It must return in a narrow boundary current where the relative vorticity can be strong (lots of horizontal shear). (PV balance is Coriolis and relative vorticity.)
- Is the boundary current on the western side or the eastern side?

Does the ocean look like this?

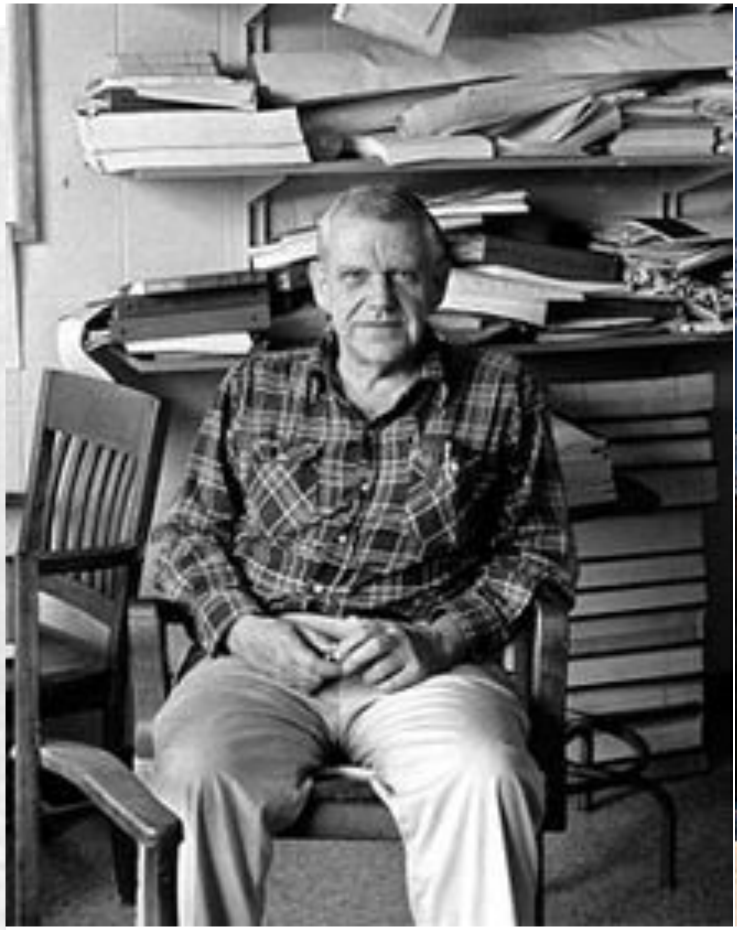


Or this?

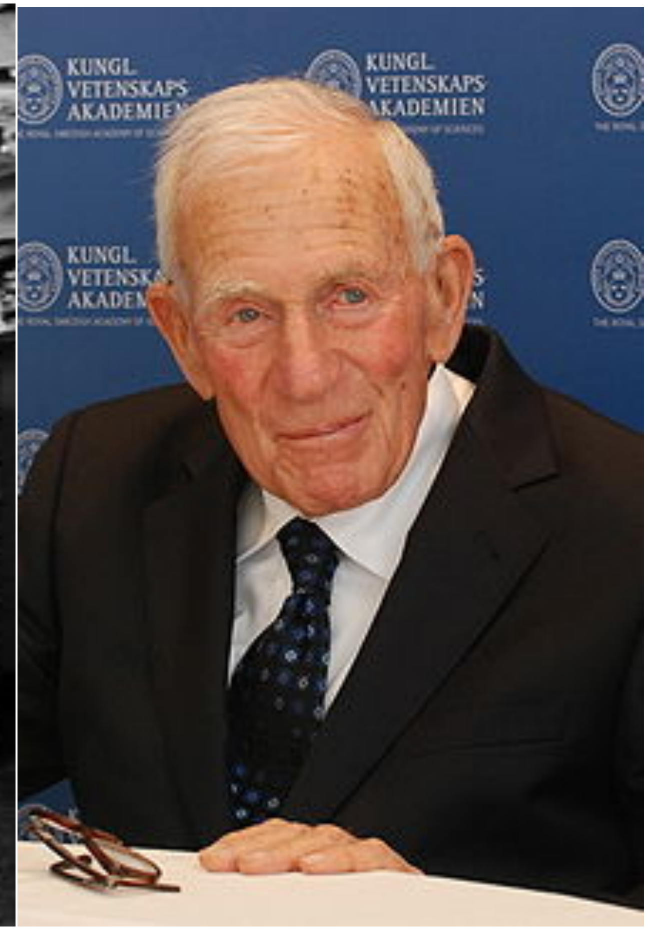


Sverdrup, Stommel, and Munk

Harald Sverdrup



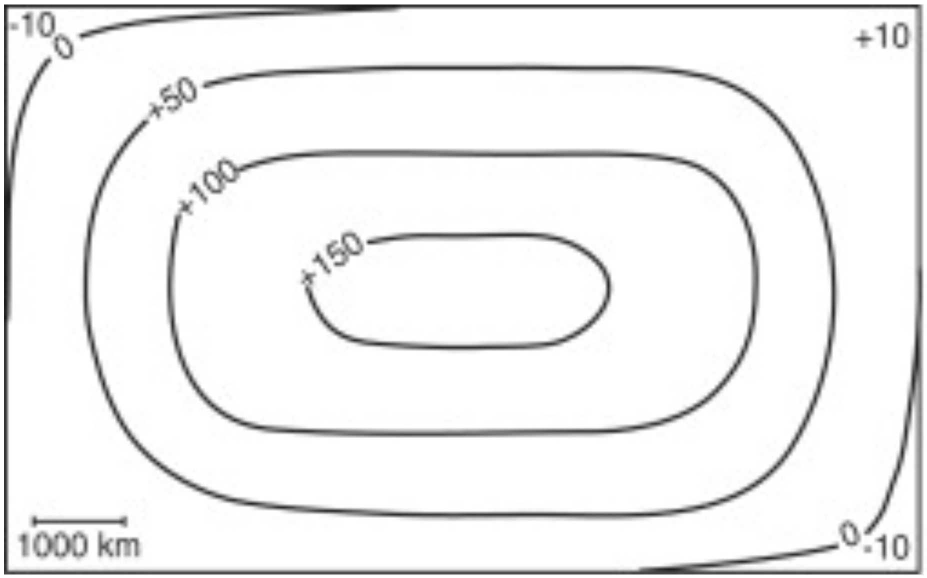
Henry Stommel

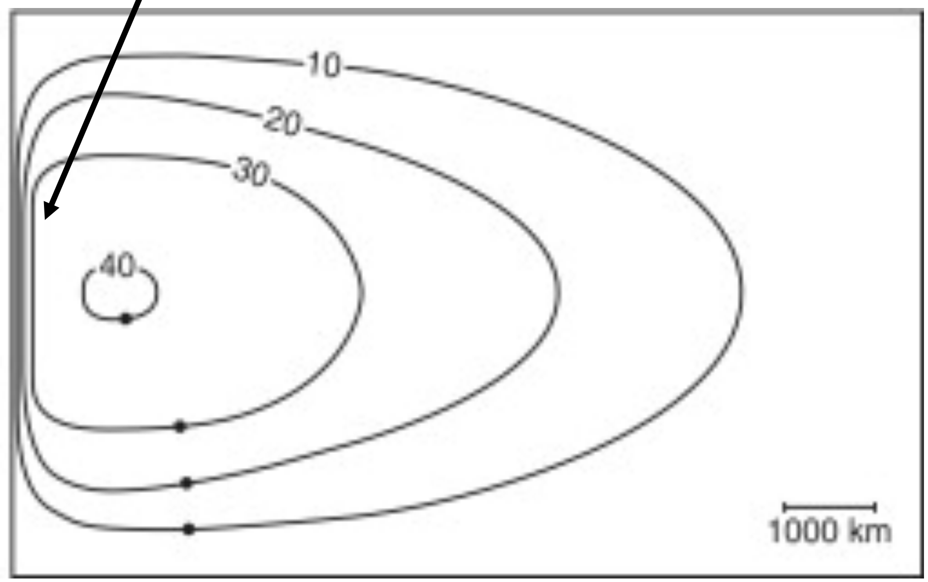


Walter Munk

Stommel's wind driven circulation

Faster= more friction to counter changes in f



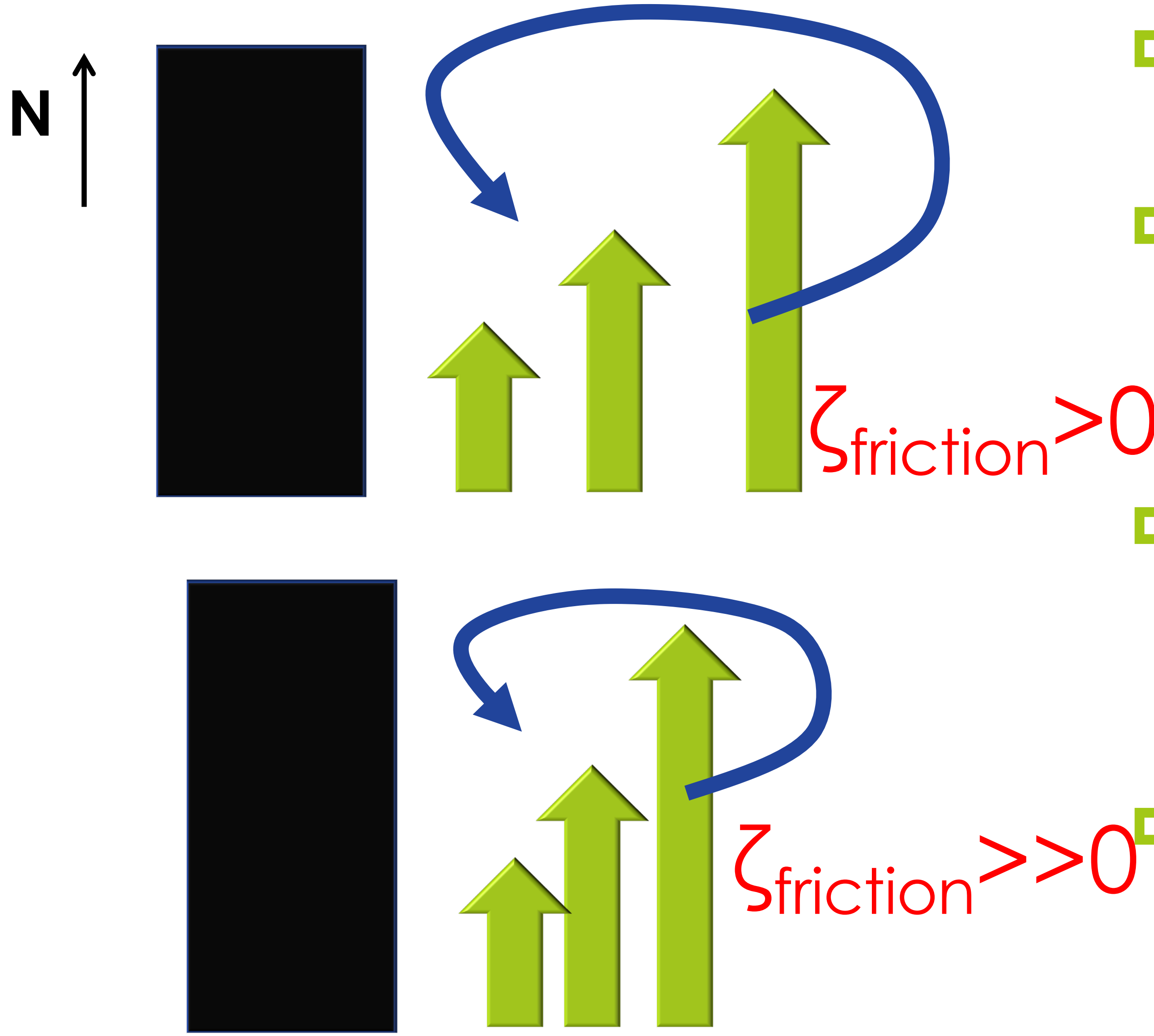


Theoretical rotating disk
(ie no variations in f)

Theoretical rotating globe
(variations in f)

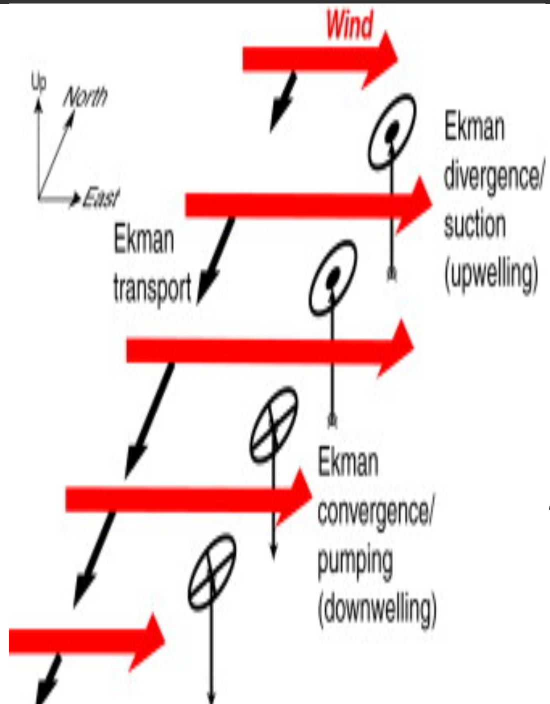
Bottom friction needed to get rid of extra vorticity from wind
but wind driven circulation doesn't reach bottom!

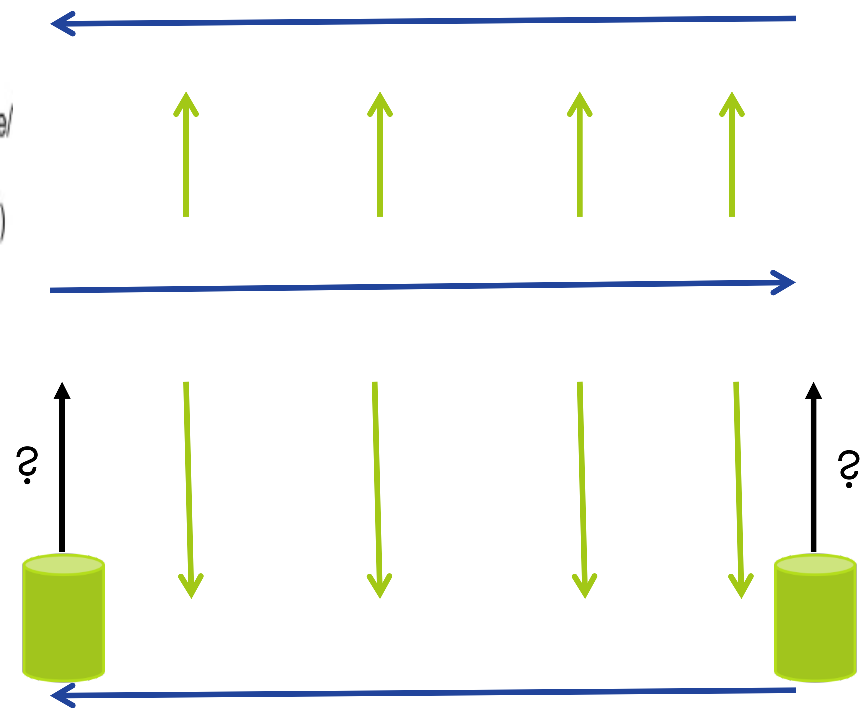
Munk: Costal Friction



- Need to consider friction
- Friction can drives horizontal shear and relative vorticity
- The greater the horizontal shear, the larger the relative vorticity
- Right hand rule to determine if ζ is positive or negative

Sverdrup + Stommel + Munk





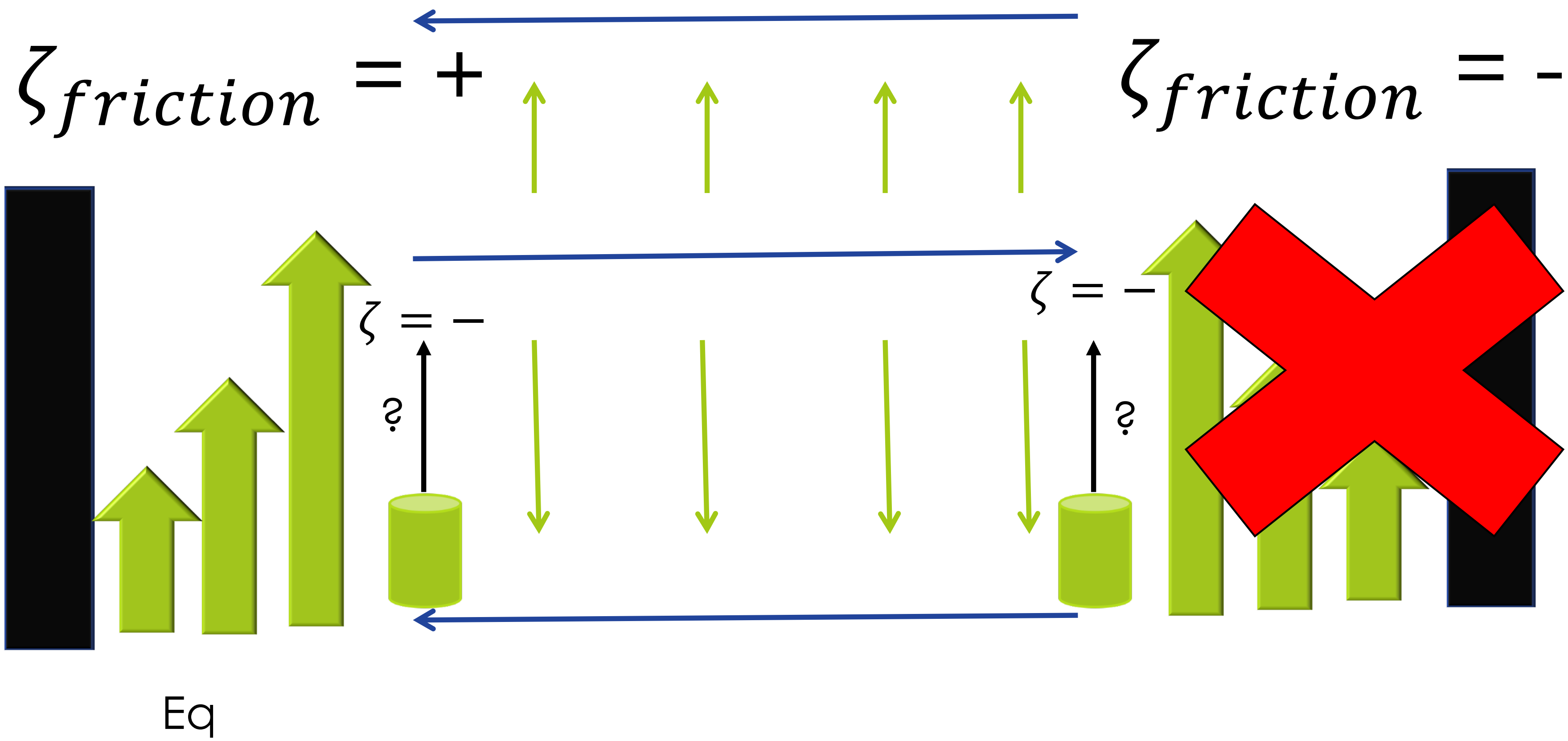
Sverdrup Transport North

Sverdrup Transport south

ζ decreasing = increased clockwise vorticity?

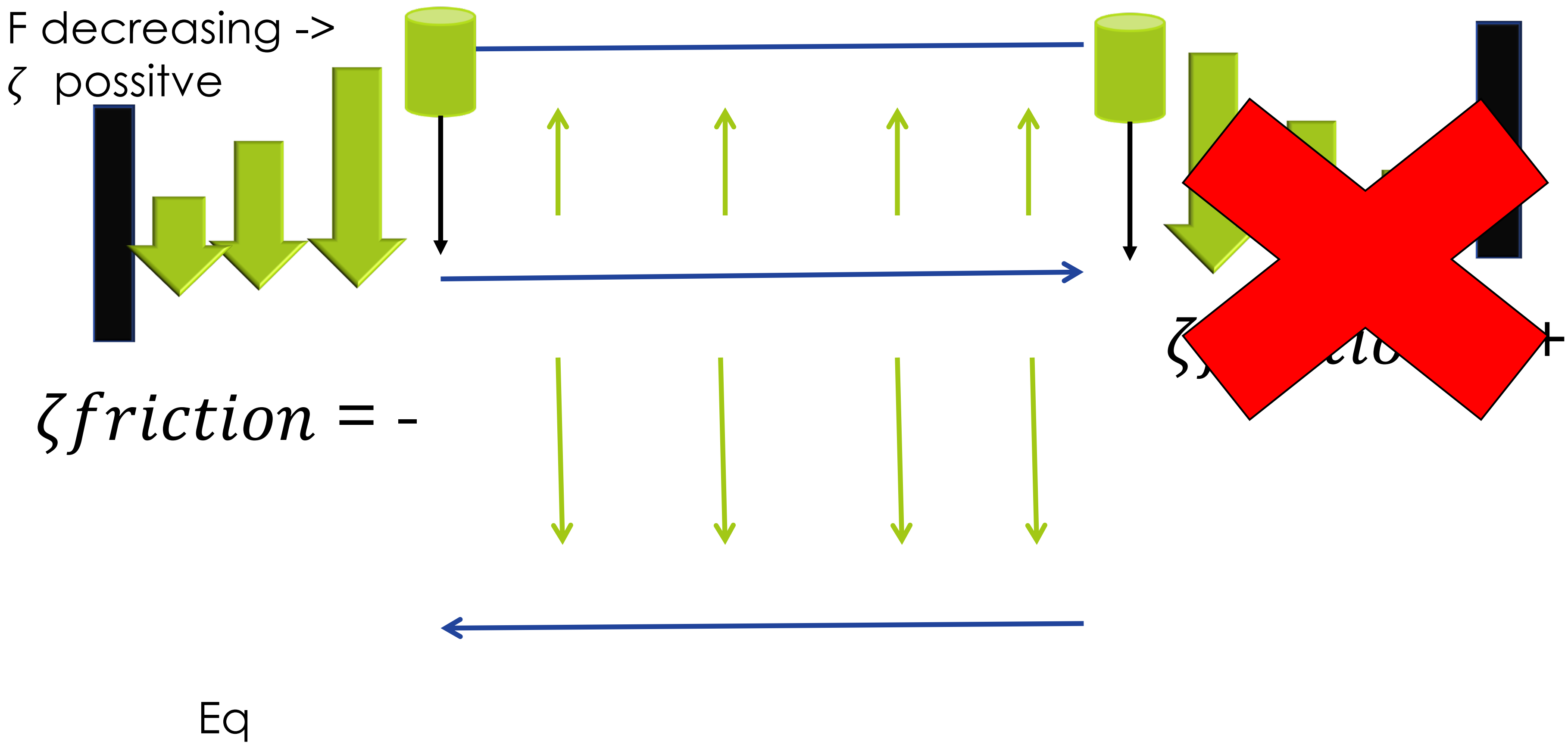
$$Q = Constant = \frac{f + \zeta}{h}$$

Sverdrup + Stommel + Munk



- Particle moves north, increase f, driving a negative (clockwise) vorticity in the particle.
- External force (friction) comes in to remove that vorticity. Need positive relative vorticity from friction – can only happen on western boundary

Sverdrup + Stommel + Munk

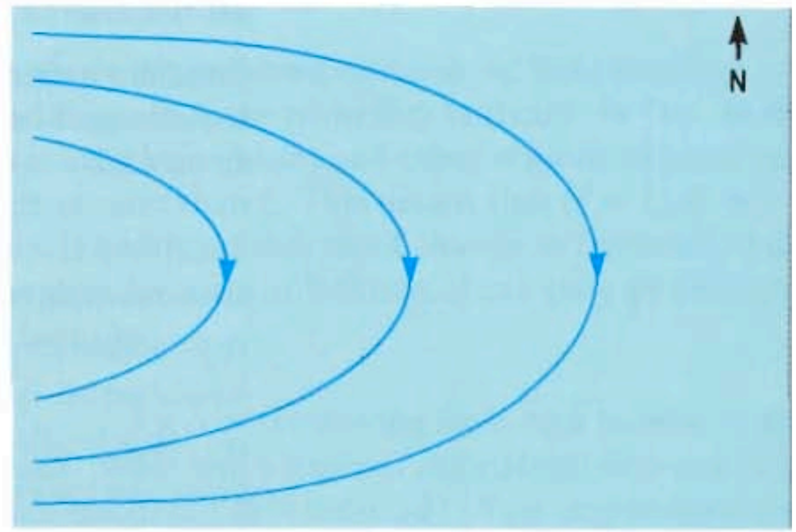


$$Q = Constant = \frac{f + \zeta}{h}$$

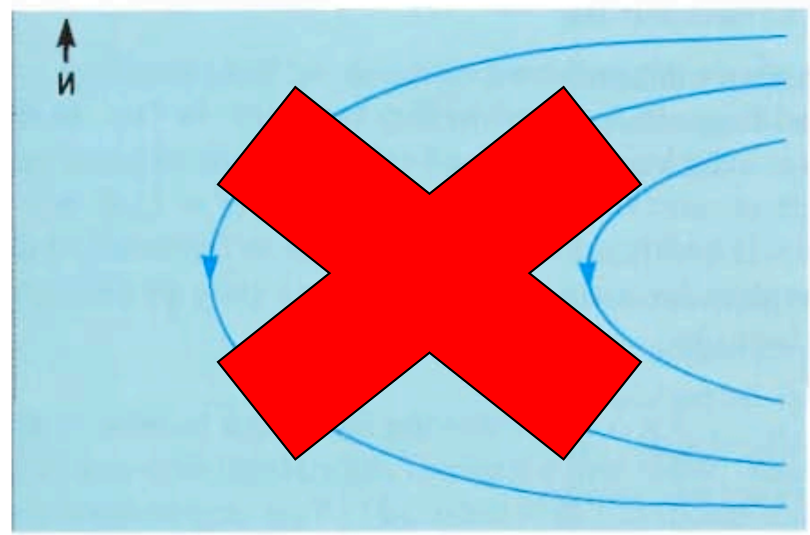
Sverdrup + Stommel + Munk

- A viscous region is necessary to remove this vorticity – must be a narrow region next to a boundary
- Viscous boundary current puts in the opposite type of relative vorticity
- Can it be on ANY boundary?
- NO – has to be on a western boundary!
  - B/c: Ekman downwelling: wind puts in negative vorticity in gyre
  - B/c: WBC for downwelling gyre must put in positive vorticity

Does the ocean look like this?



Or this?



Ship Drift and Currents

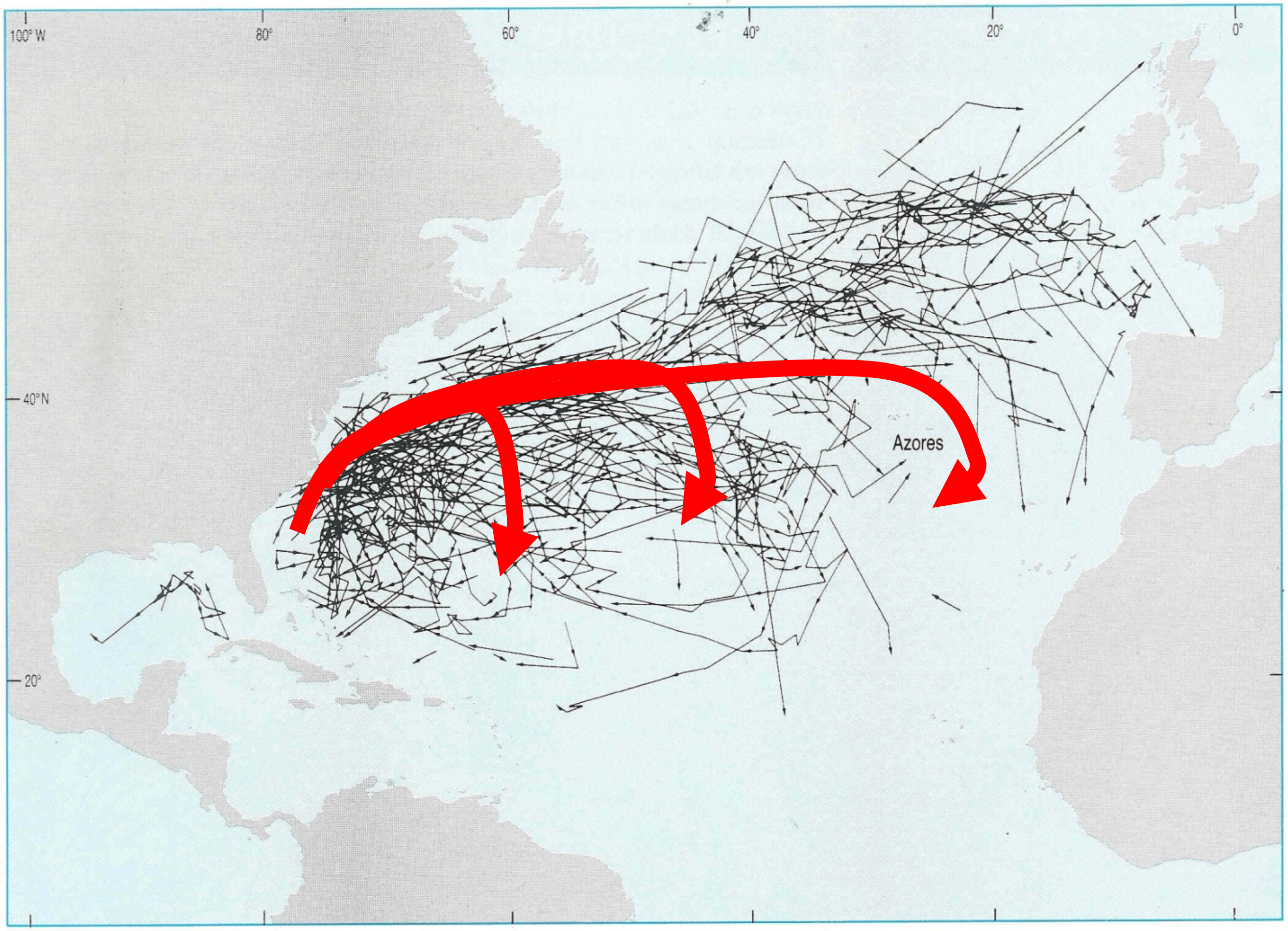


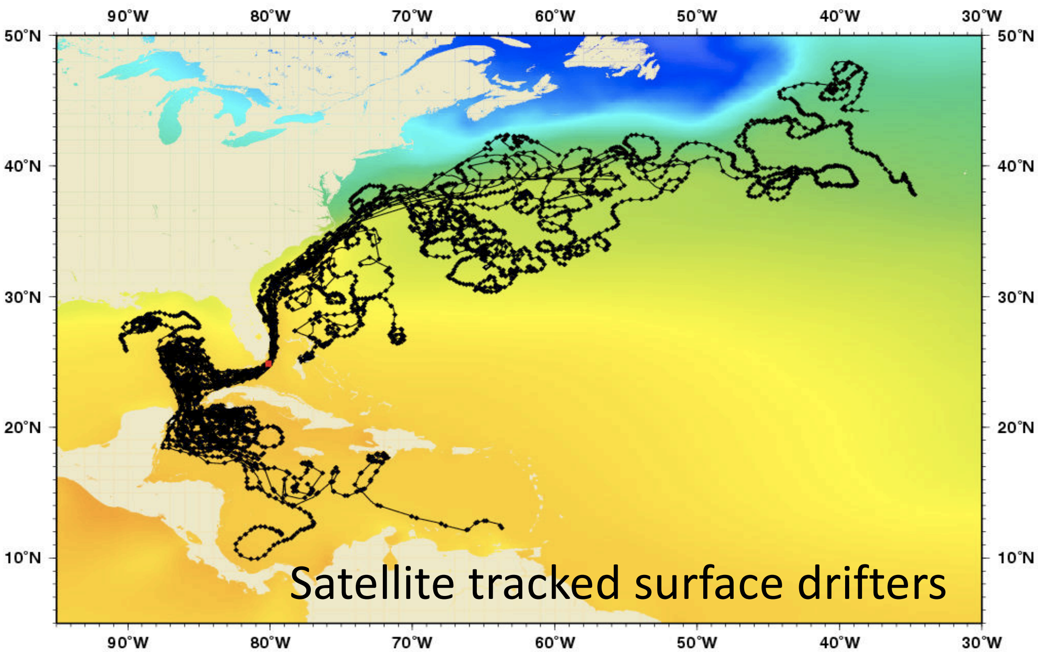
Figure 3.2 The paths of drifting derelict sailing vessels (and a few drifting buoys) over the period 1883–1902. This chart was produced using data from the monthly Pilot Charts of the US Navy Hydrographic Office. The paths are extremely convoluted and cross one another, but the general large-scale anticyclonic circulation of the North Atlantic may just be distinguished.

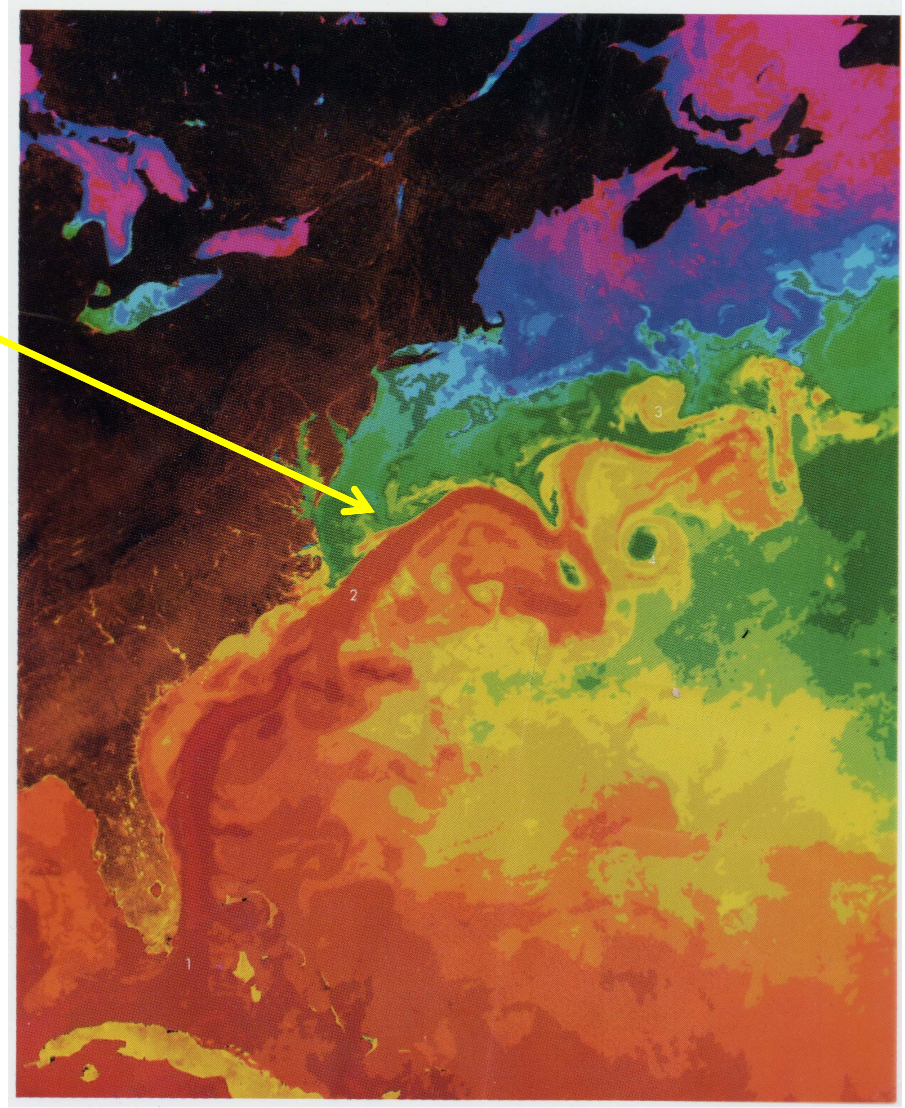
Richardson, Science (1980)

Ship drift data from1883-1902 shows clock-wise surface circulation of the North Atlantic

The Gulf Stream

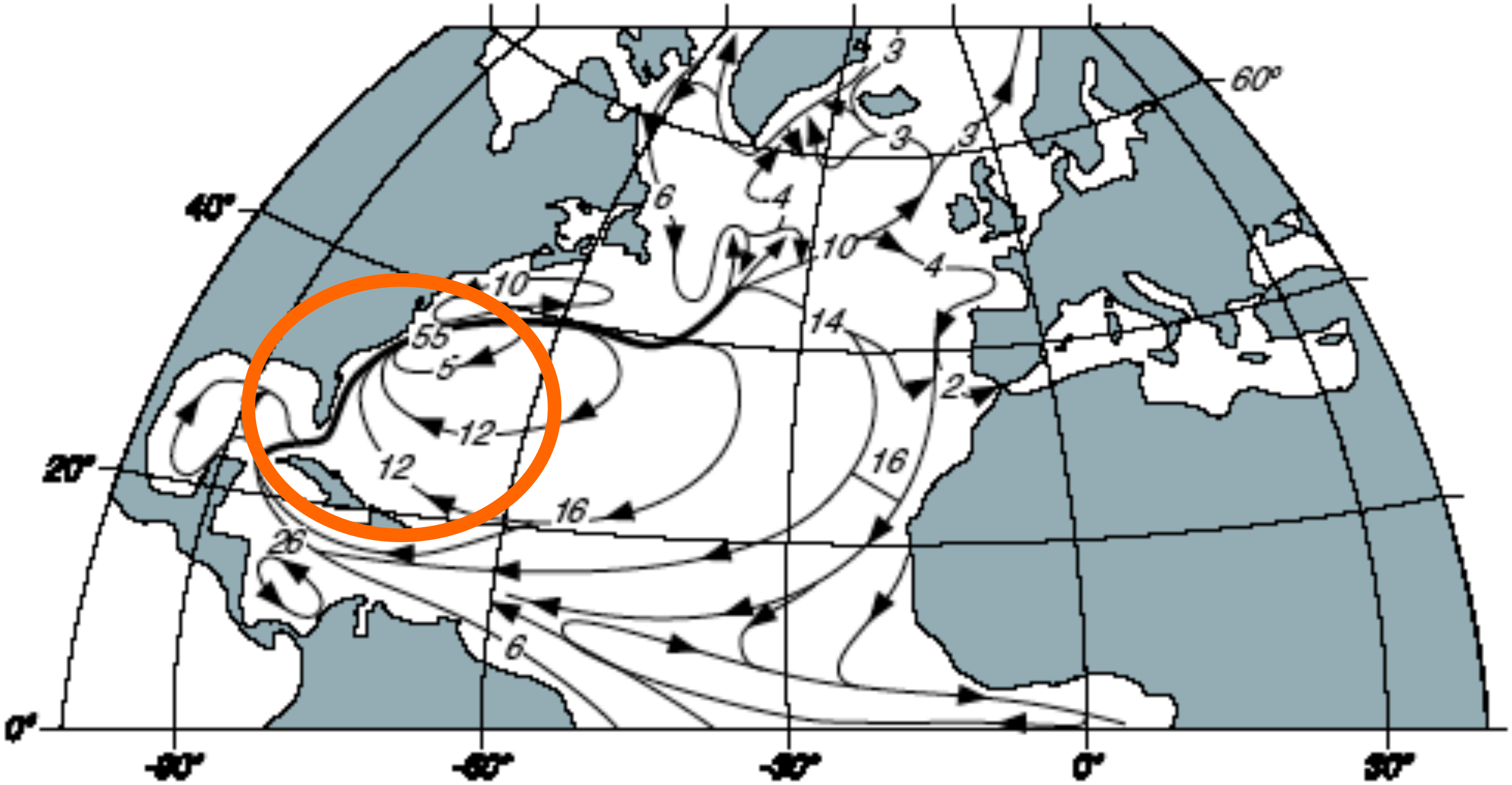
The swift western boundary current of the North Atlantic, *a sign of western intensification.*





Like the atmosphere, fast current are located where isopynals are close together
Strong currents generate instabilities in the form of eddies.

The Gulf Stream



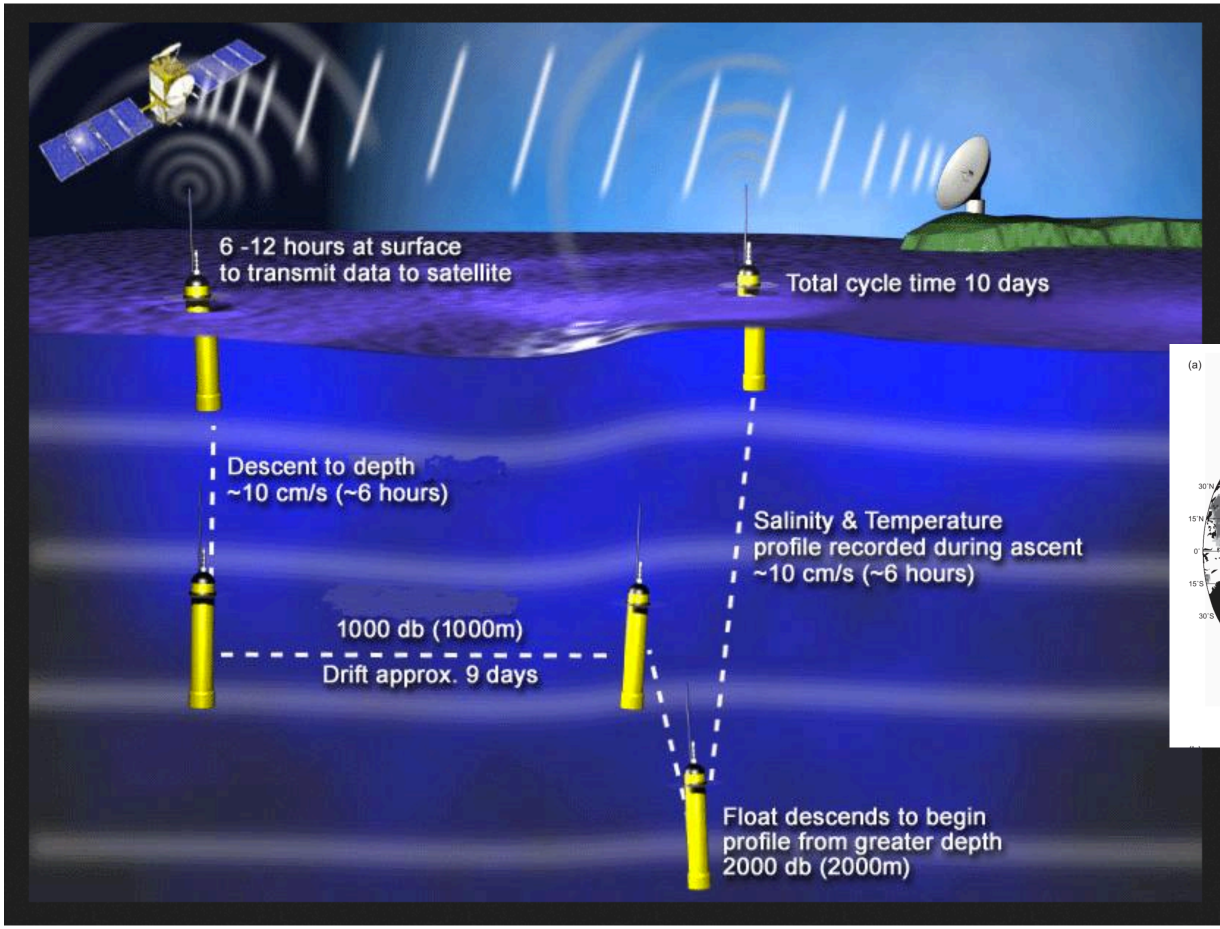
- North Atlantic currents and their volume transport in units of Sv (10^6 m^3/s).

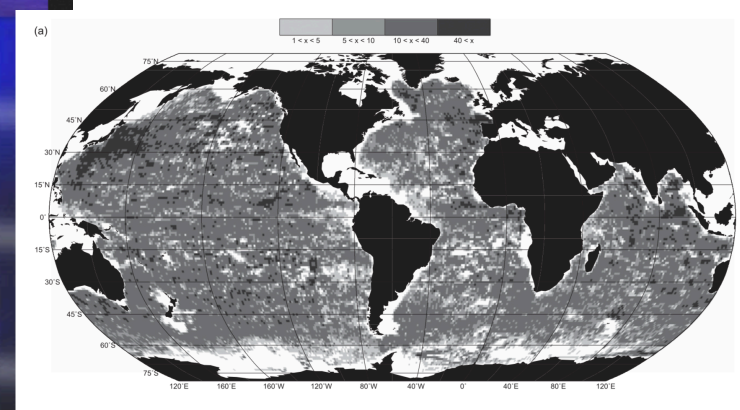
After Sverdrup, Johnson, and Fleming (1942)

- Gulf stream current is equivalent to about 30-150 Sv
- Unit of measure is Sverdrup (Sv) - 1 Sv = 10^6 m^3 s^{-1}= 5 million bathtubs a second!

*All the rivers in the world are equivalent to about 1 Sv

Theory vs observations?





(Gray and Riser, 2014)

Theory vs observations?

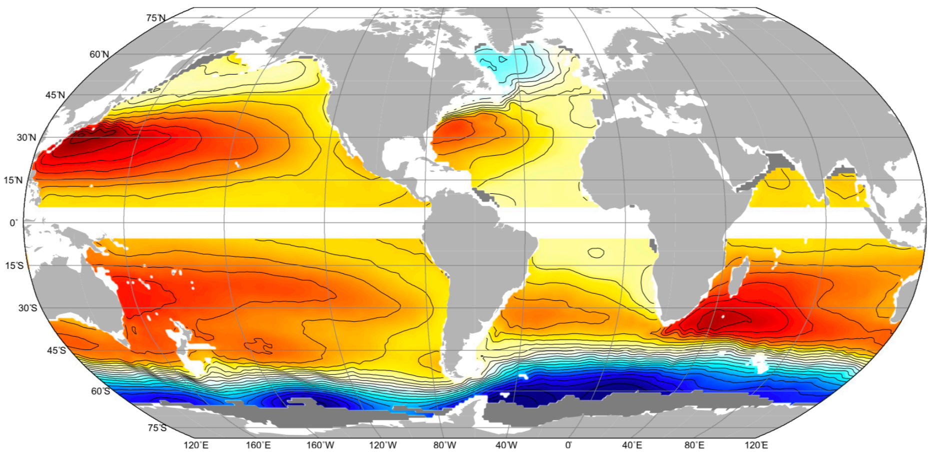
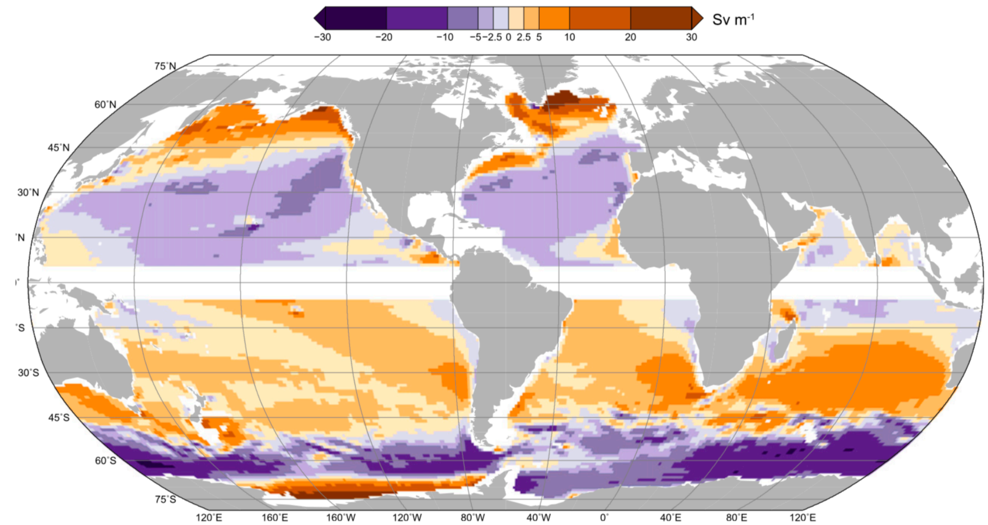


FIG. 3. Mean absolute geostrophic streamfunction at 200 db from Argo data for December 2004–November 2010. Contour interval is 10 dyn cm. Colors as in Fig. 2.



IG. 6. Mean wind-derived transport ($V_{Sv} - V_E$) from QuikSCAT [Sv m^{-1}; where 1 Sverdrup (Sv) ≡ 10^6 m^3 s^{-1}], averaged over August 1999–October 2009. Positive (negative) values indicate northward (southward) transport.

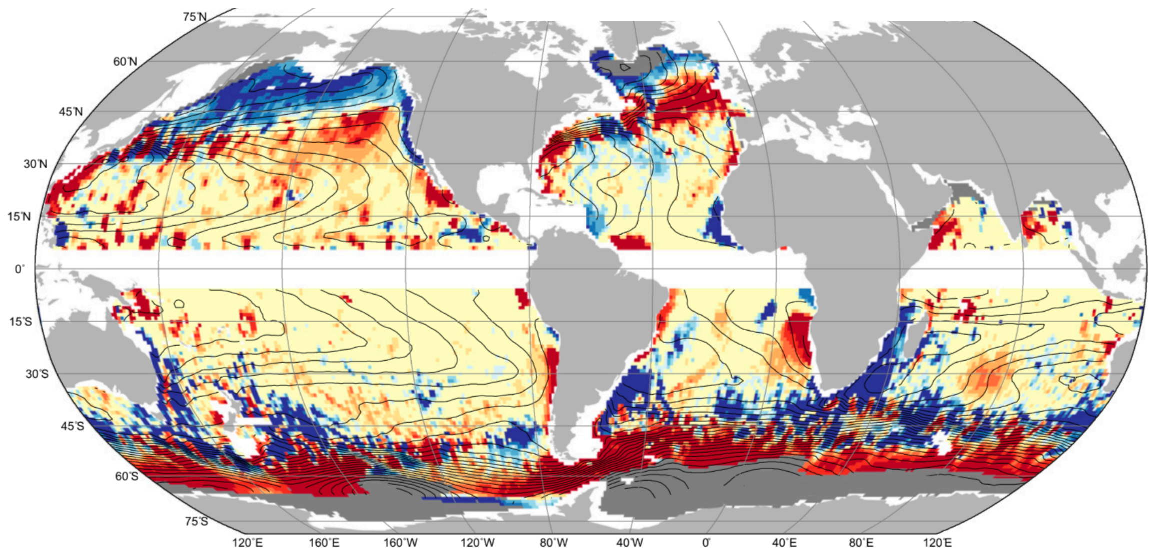
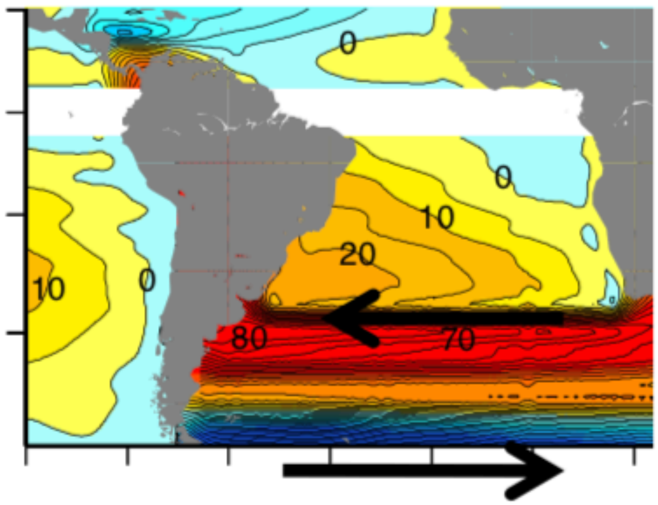
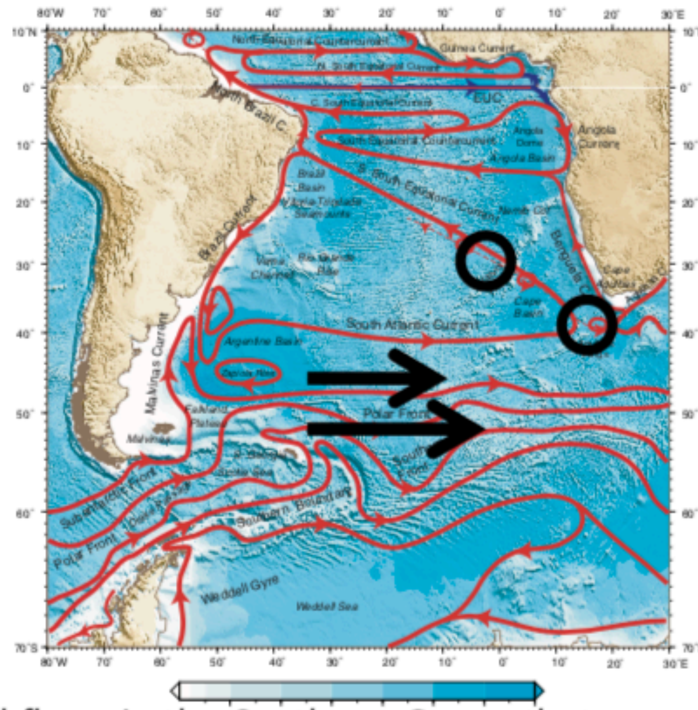


FIG. 7. Normalized difference Δ between V_g and the wind-derived transport, as defined by (10). The transport V_g is computed with h given by the depth of σ_θ 26.24, 27.24, and 27.25 for the North Pacific, Southern Hemisphere and north Indian, and North Atlantic basins. The value of Δ shown here is the min difference taking into account the uncertainty on V_g, with yellow indicating exact agreement. Areas where the given isopycnals were not present in the mean are shown in dark gray. The mean 5-db geostrophic streamfunction is contoured in black at 10–dyn cm intervals.

(Gray and Riser, 2014)

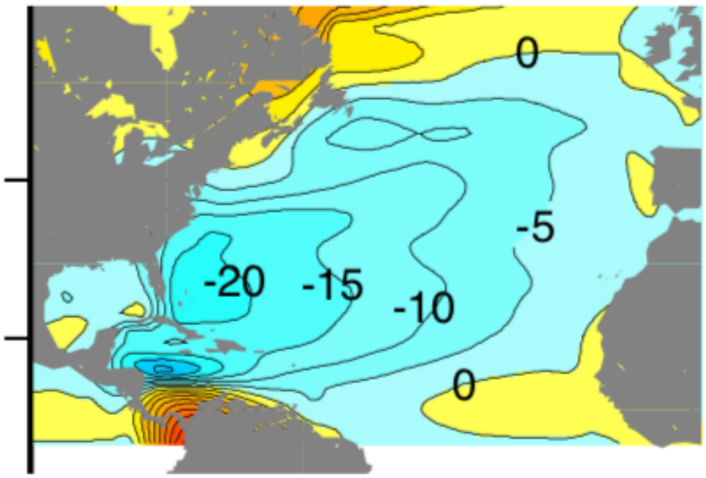
Deficiencies in the theory: Sverdrup transport and actual total transport

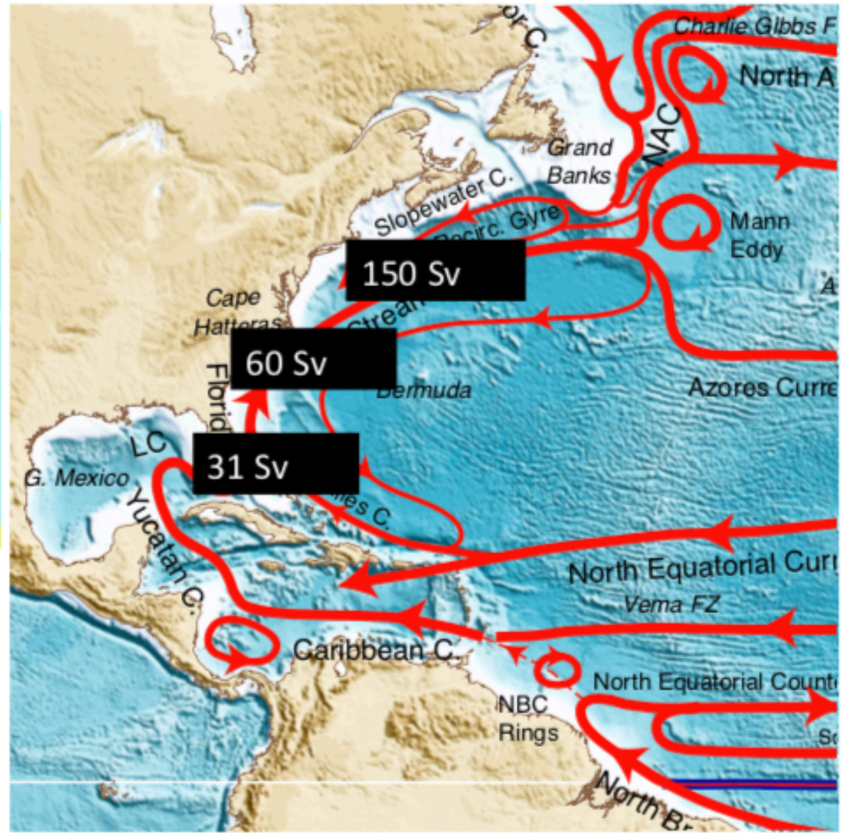




Sverdrup balance also suggests strong zonal flows in the Southern Ocean that are not observed (Agulhas from Indian Ocean does not jet westward to South America, but breaks up into eddies that move westward into Atlantic)

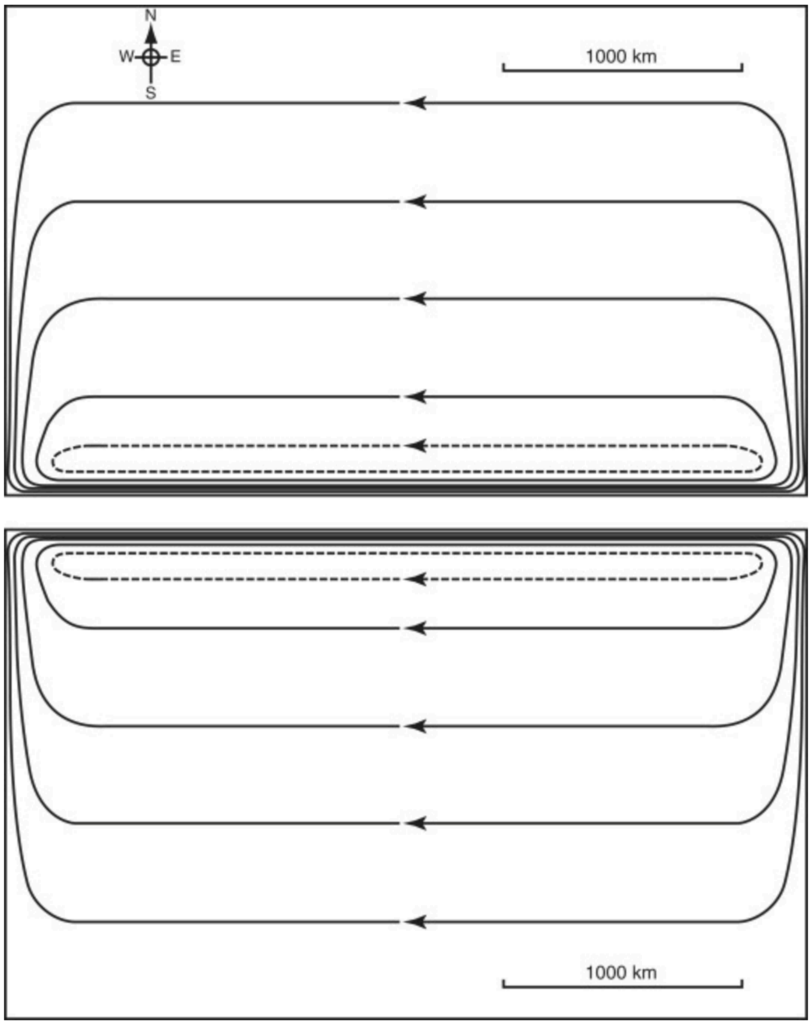
Deficiencies in the theory: Sverdrup transport and actual total transport





Sverdrup transport generally underestimates subtropical gyre WBC transports, especially after their "separation points", where their dynamics becomes "inertial", not governed by Sverdrup dynamics

Fofonoff's Model



Fofonoff: No wind, no friction, just β

-can find solution with purely zonal flow in interior and swift boundary currents using conservation of potential vorticity