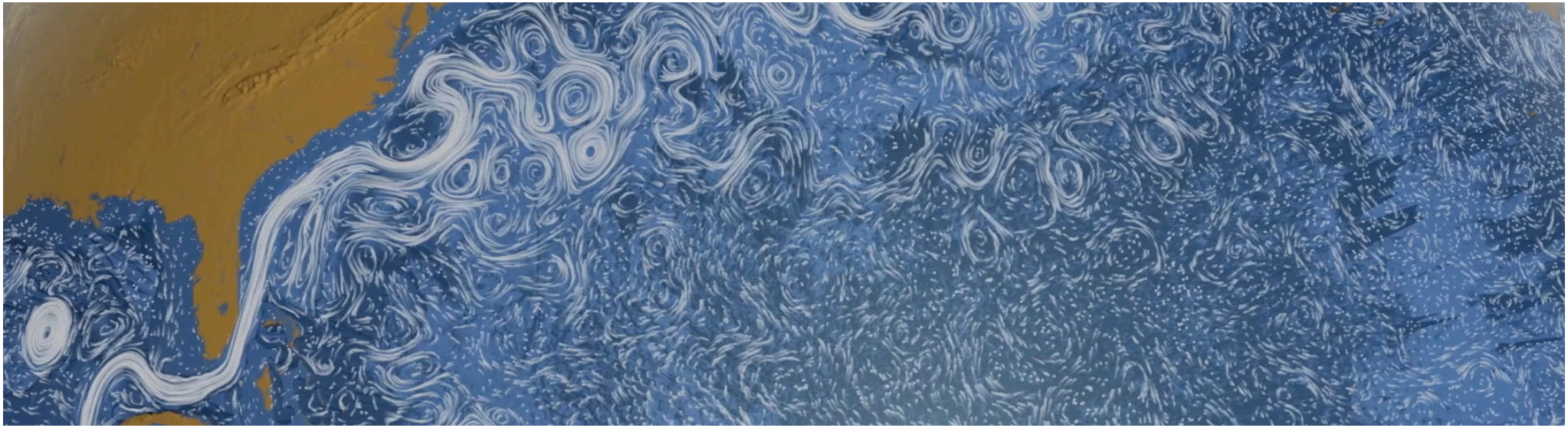


SIO 210 Dynamics III: Earth's rotation

Earth reference frame

Geoid

Rotation definitions

Centrifugal force

Coriolis force

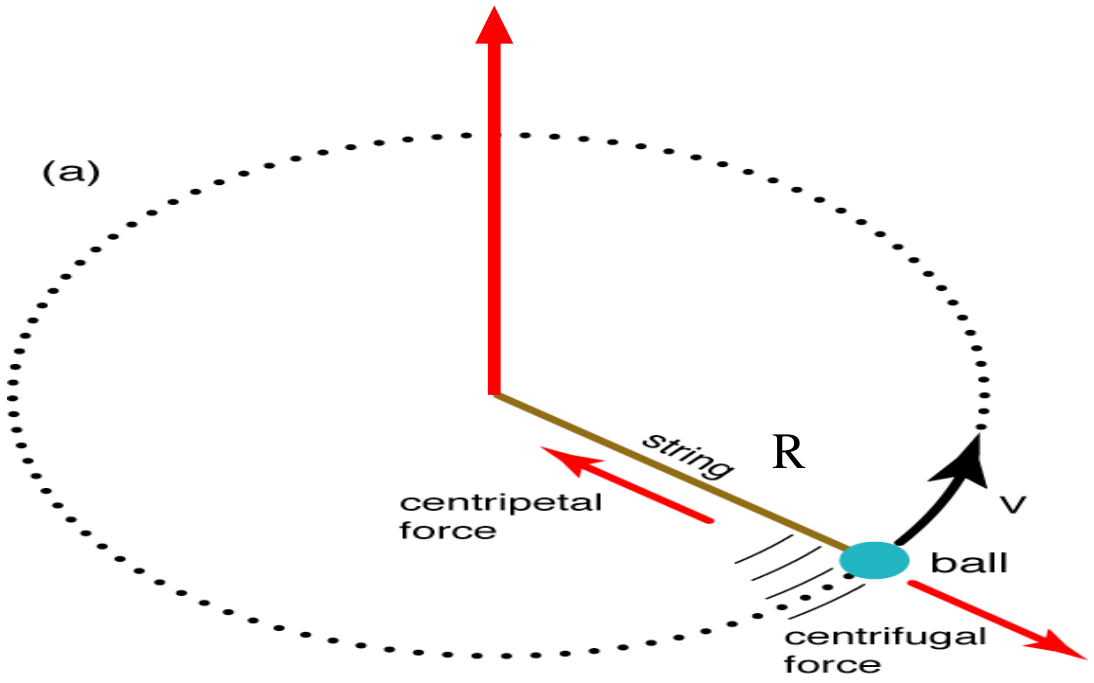
Inertial motion

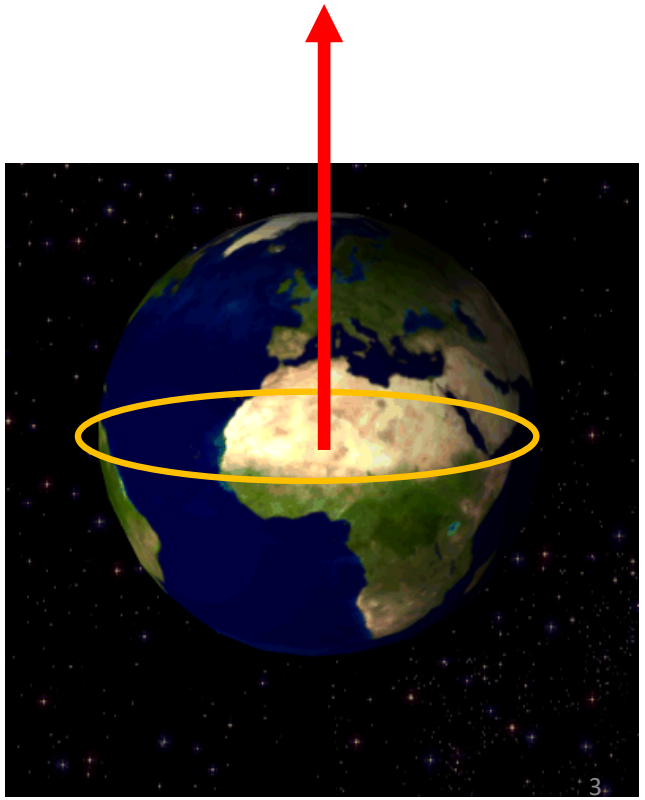
- Reading
  - DPO: Chapter 7.2.3, 7.5.1

Rotating Earth

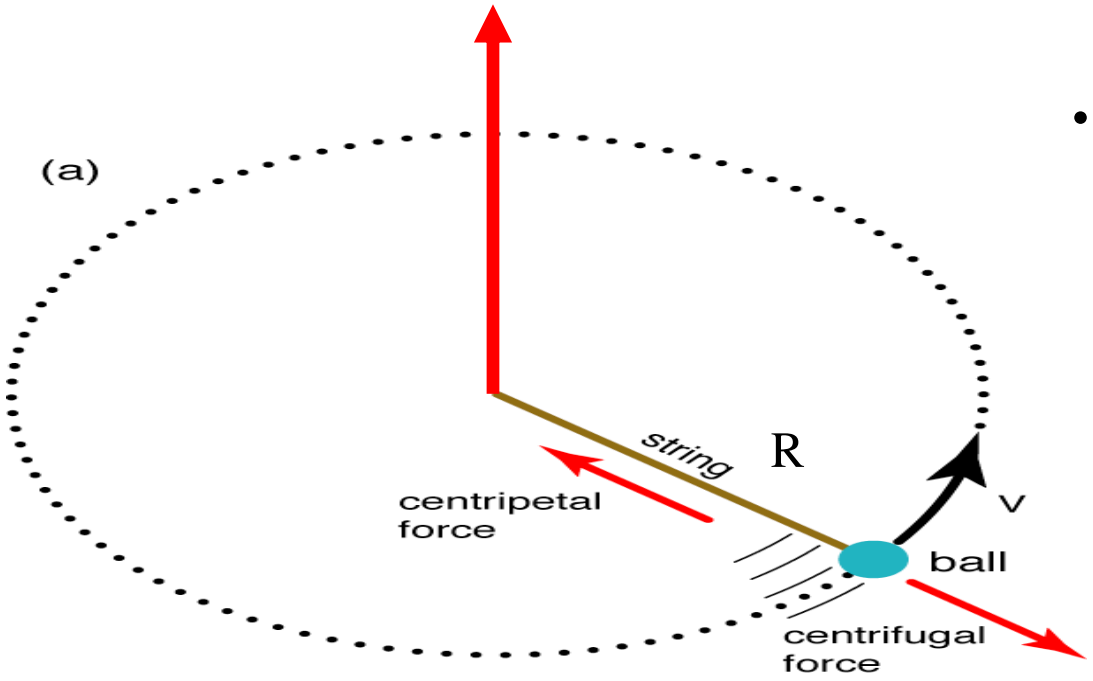
Rotating coordinates

- Vector that expresses direction of rotation and how fast it is rotating:
  vector pointing in direction of thumb using right-hand rule, curling fingers in direction of rotation





Rotating coordinates



- We measure things relative to Earth's "rotating reference frame".
- Quantity that tells how fast something is rotating:

  Angular speed or angular velocity

  Ω = angle/second

  360° is the whole circle, but express angle in radians (2π radians = 360°)

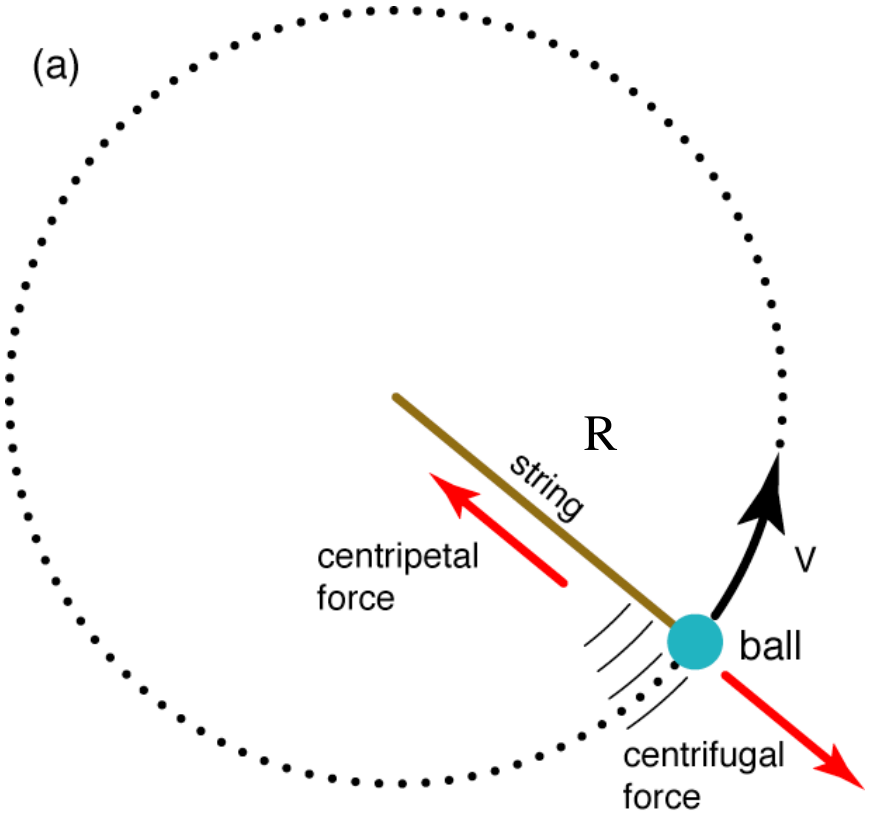
  For Earth: $\Omega = 2\pi$ / 1 day = 2π / 86,400 sec = 0.707×10^{-4} /sec

  Also $\Omega = v/R$ where v is the measured velocity and R is the radius to the axis of rotation (therefore $v = \Omega R$)

At home: calculate speed you are traveling through space if you are at the equator (radius of earth is about 6371 km), then calculate it at 30N as well. (Do some geometry to figure out the distance from the axis at 30N.)

Centripetal and Centrifugal forces

(now looking straight down on the rotating plane)



Centripetal force is the actual force that keeps the ball "tethered" (here it is the string, but it can be gravitational force)

Centrifugal force is the pseudo-force (apparent force) that one feels due to lack of awareness that the coordinate system is rotating or curving

centrifugal acceleration = $\Omega^2 R$ (outward) (Units of m/sec^2)

Effect of centrifugal force on ocean and earth

Centrifugal force acts on the ocean and earth. It is pointed outward away from the rotation axis.

Therefore it is maximum at the equator (maximum radius from axis) and minimum at the poles (0 radius).

$\Omega = 0.707 \times 10^{-4}$ /sec

At the equator, R ~ 6380 km

→ $\Omega^2 R = .032$ m/s^2

Compare with gravity = 9.8 m/s^2

Centrifugal force should cause the equator to be deflected (0.032/9.8) x 6380 km = 21 km outward compared with the poles.
(i.e. about 0.3%)

Effect of centrifugal force on ocean and earth

Radius:

Equatorial 6,378.135 km

Polar 6,356.750 km

Mean 6,372.795 km

(From wikipedia entry on Earth)

The ocean is not 20 km deeper at the equator, rather the earth itself is deformed!

We bury the centrifugal force term in the gravity term (which we can call “effective gravity”), and ignore it henceforth. Calculations that require a precise gravity term should use subroutines that account for its latitudinal dependence.

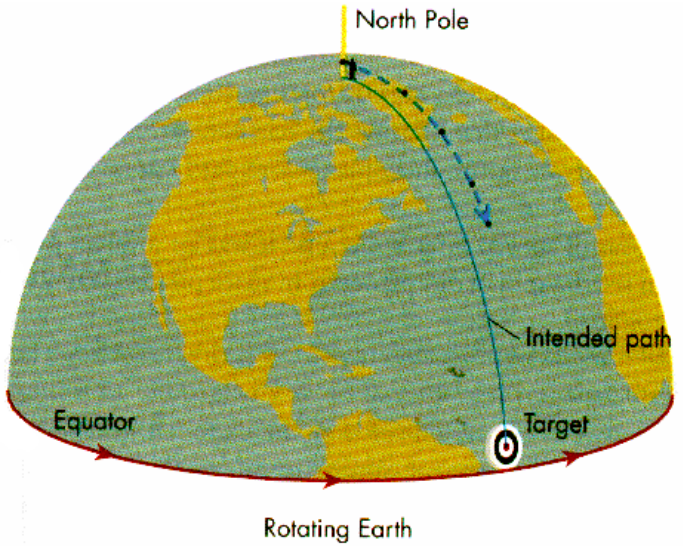
Coriolis effect

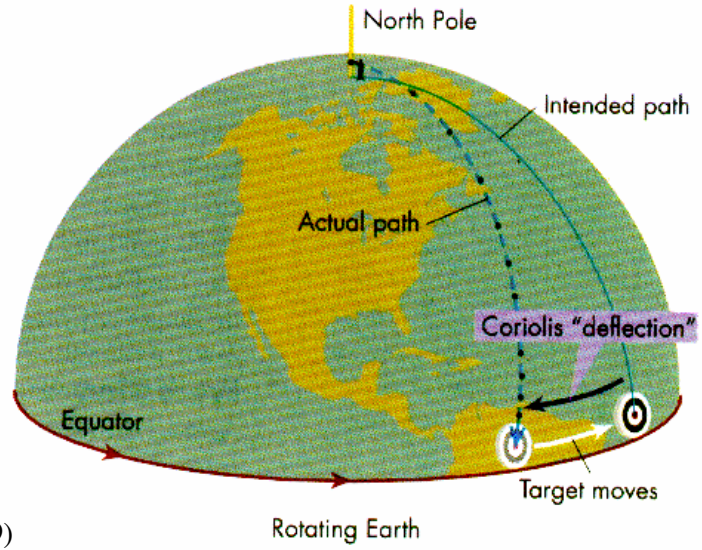
Inertial motion: (Newton's 1st law)
motion in a straight line relative to the fixed stars

Coriolis effect: What does that straight, inertial line look like from our rotating frame of reference?

It looks like it is mysteriously deflecting, contrary to 1st law! But we are the ones who are moving (rotating)

Coriolis effect deflects bodies (water parcels, air parcels) to the:
RIGHT in the northern hemisphere
LEFT in the southern hemisphere





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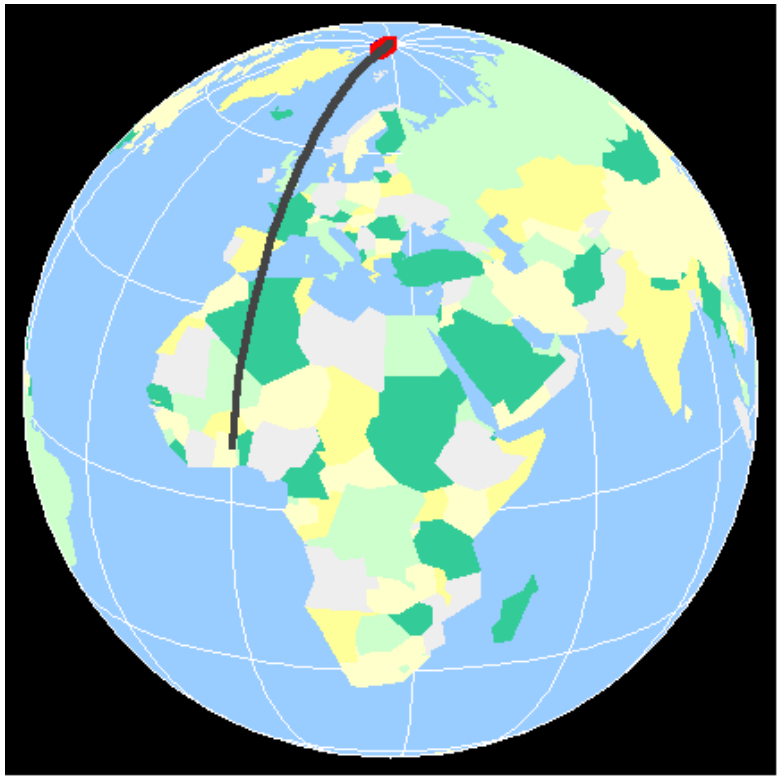


http://physics.oregonstate.edu/~mcintyre/coriolis/North_Pole_GIF.html

Coriolis effect

Look online for countless animations

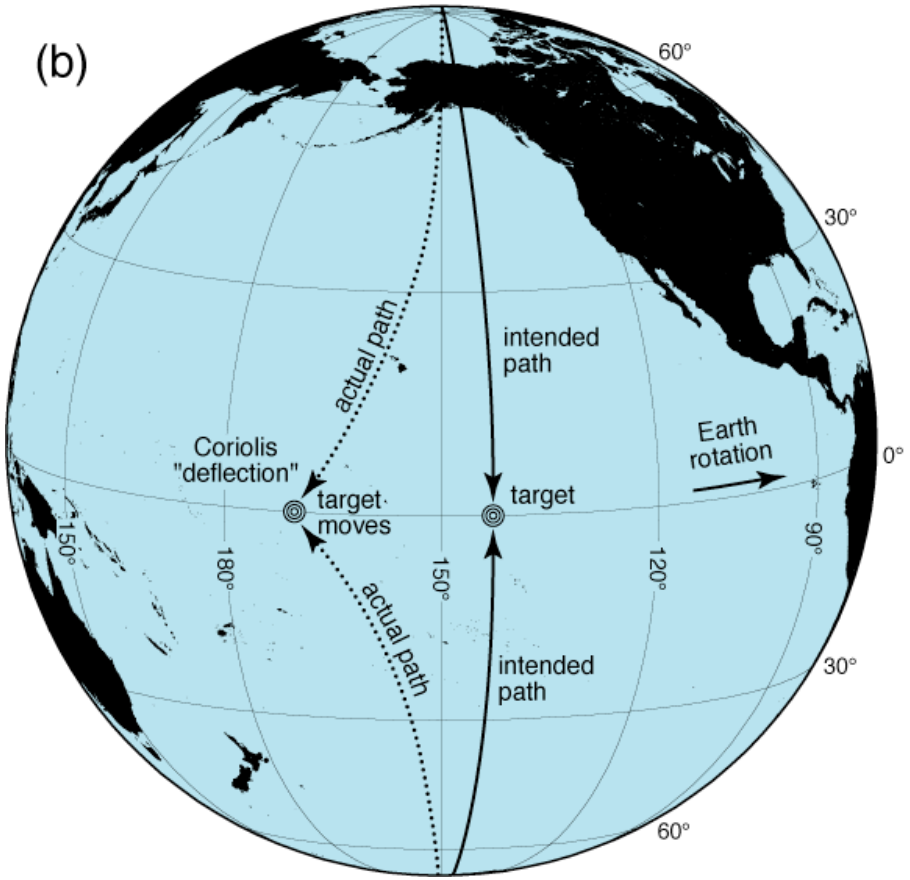
Comprehensive and pedagogical:
http://www.physics.orst.edu/~mcintyre/coriolis/index.html

Khan academy on hurricanes
https://www.khanacademy.org/partner-content/nova/clouds/v/hurricanes

Youtube video demonstrations
https://www.youtube.com/watch?v=mcPs_OdQOYU

https://www.youtube.com/watch?v=aeY9tY9vKgshttps://www.youtube.com/watch?v=i2mec3vgeaI

Coriolis force



Additional terms in x, y momentum equations, at latitude φ
(horizontal motion is much greater than vertical)

x-momentum equation:

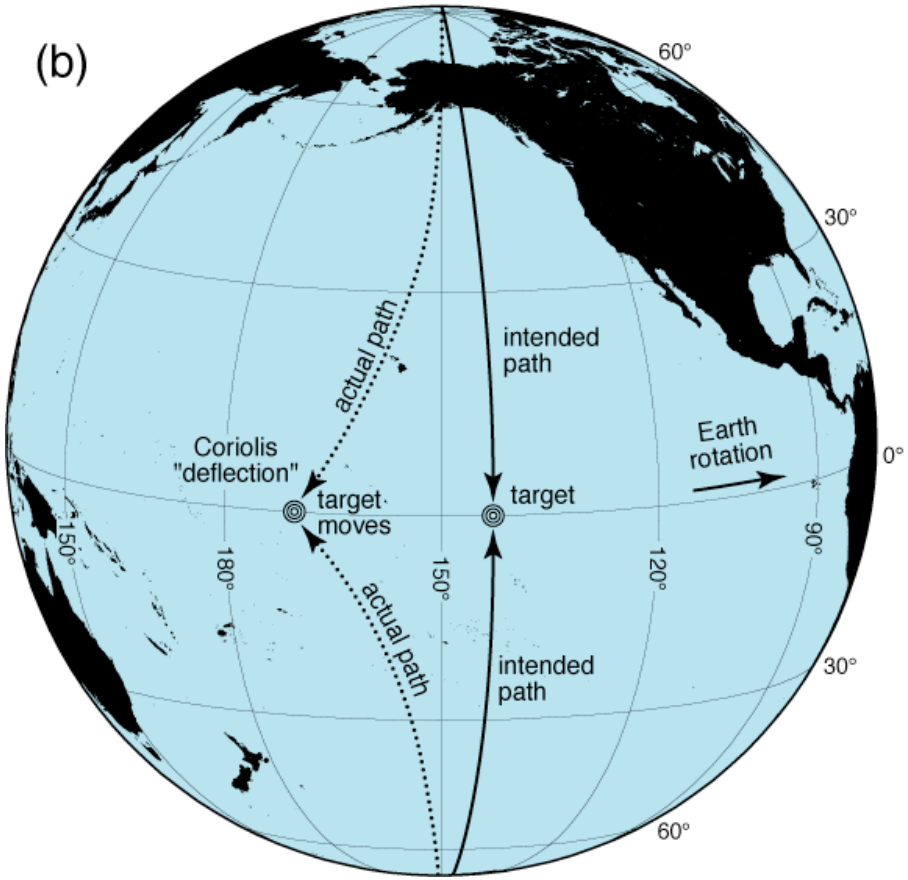
$$-2\,\Omega\sin\varphi\, v = -f\, v$$

y-momentum equation:

$$2\,\Omega\sin\varphi\, u = f\, u$$

$f = 2\,\Omega\sin\varphi$ is the "Coriolis parameter". It depends on latitude (projection of total Earth rotation on local vertical)

Coriolis parameter



$f = 2\Omega\sin\varphi$ is the "Coriolis parameter"

$2\Omega = 1.458 \times 10^{-4}$/sec

At equator (φ=0, $\sin\varphi$=0):
$f = 0$

At 30° N (φ=30° , $\sin\varphi$=0.5):
$f = 0.729 \times 10^{-4}$/sec

At north pole (φ=90° , $\sin\varphi$=1):
$f = 1.458 \times 10^{-4}$/sec

At 30° S (φ=-30° , $\sin\varphi$=-0.5):
$f = -0.729 \times 10^{-4}$/sec

At south pole (φ=-90° , $\sin\varphi$=-1):
$f = -1.458 \times 10^{-4}$/sec

Complete force (momentum) balance with rotation

Three equations in local Cartesian coordinates:

Horizontal (x) (west->east)

acceleration + advection + Coriolis = pressure gradient force + viscous term

Horizontal (y) (south->north)

acceleration + advection + Coriolis = pressure gradient force + viscous term

Vertical (z) (down->up)

acceleration +advection (+ neglected very small Coriolis) = pressure gradient force + effective gravity (including centrifugal force) + viscous term

Complete equations of motion in rotating coordinates

(momentum equations in Cartesian coordinates)

x: $\partial u/\partial t + u\,\partial u/\partial x + v\,\partial u/\partial y + w\,\partial u/\partial z - fv =$

$$- (1/\rho)\partial p/\partial x + \partial/\partial x(A_H \partial u/\partial x) + \partial/\partial y(A_H \partial u/\partial y) + \partial/\partial z(A_V \partial u/\partial z) \qquad (7.11a)$$

y: $\partial v/\partial t + u\,\partial v/\partial x + v\,\partial v/\partial y + w\,\partial v/\partial z + fu =$

$$- (1/\rho)\partial p/\partial y + \partial/\partial x(A_H \partial v/\partial x) + \partial/\partial y(A_H \partial v/\partial y) + \partial/\partial z(A_V \partial v/\partial z) \qquad (7.11b)$$

z: $\partial w/\partial t + u\,\partial w/\partial x + v\,\partial w/\partial y + w\,\partial w/\partial z$ (+ neglected small Coriolis) =

$$- (1/\rho)\partial p/\partial z - g + \partial/\partial x(A_H \partial w/\partial x) + \partial/\partial y(A_H \partial w/\partial y) + \partial/\partial z(A_V \partial w/\partial z) \qquad (7.11c)$$

(where g contains both actual g and effect of centrifugal force)

Force balance with rotation
1. Inertial motion

Inertial motion force balance

Three equations in local Cartesian coordinates:

Horizontal (x) (west->east)

acceleration + advection + Coriolis = pressure gradient force + viscous term

Horizontal (y) (south->north)

acceleration + advection + Coriolis = pressure gradient force + viscous term

Vertical (z) (down->up)

acceleration +advection (+ neglected very small Coriolis) = pressure gradient force + effective gravity (including centrifugal force) + viscous term

Force balance with rotation
1. Inertial motion

That is:

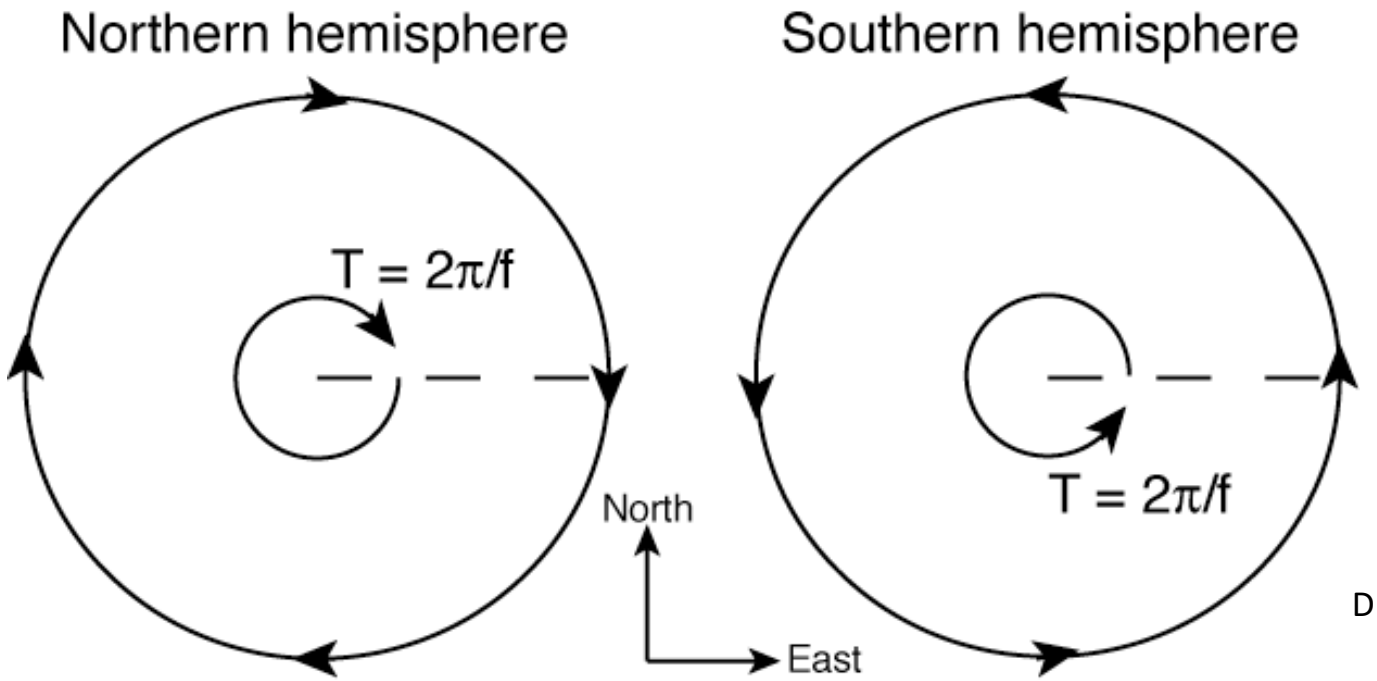
$$x: \quad \partial u/\partial t - fv = 0$$
$$y: \quad \partial v/\partial t + fu = 0$$
$$z: \quad 0 = -(1/\rho)\partial p/\partial z - g$$

To solve (if you are interested)
Solve y-eqn for u and substitute in x eqn (or vice versa).

Solutions are velocity circles:

$u = U_o \sin(ft)$ and $v = U_o \cos(ft)$ where U_o is an arbitrary amplitude

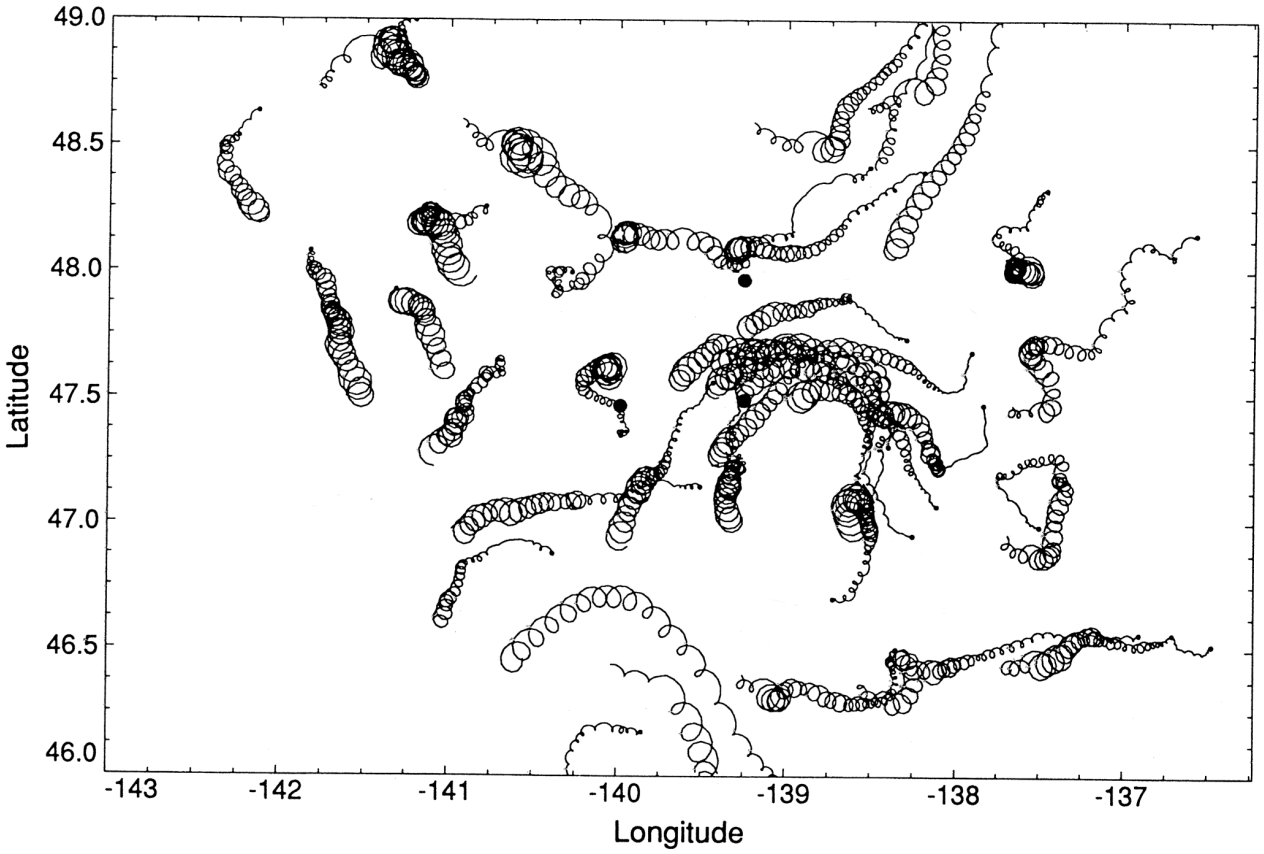
Force balance with rotation
1. Inertial motion



DPO Fig. S7.8a

Balance of Coriolis and acceleration terms: push the water and it turns to the right (NH), in circles.

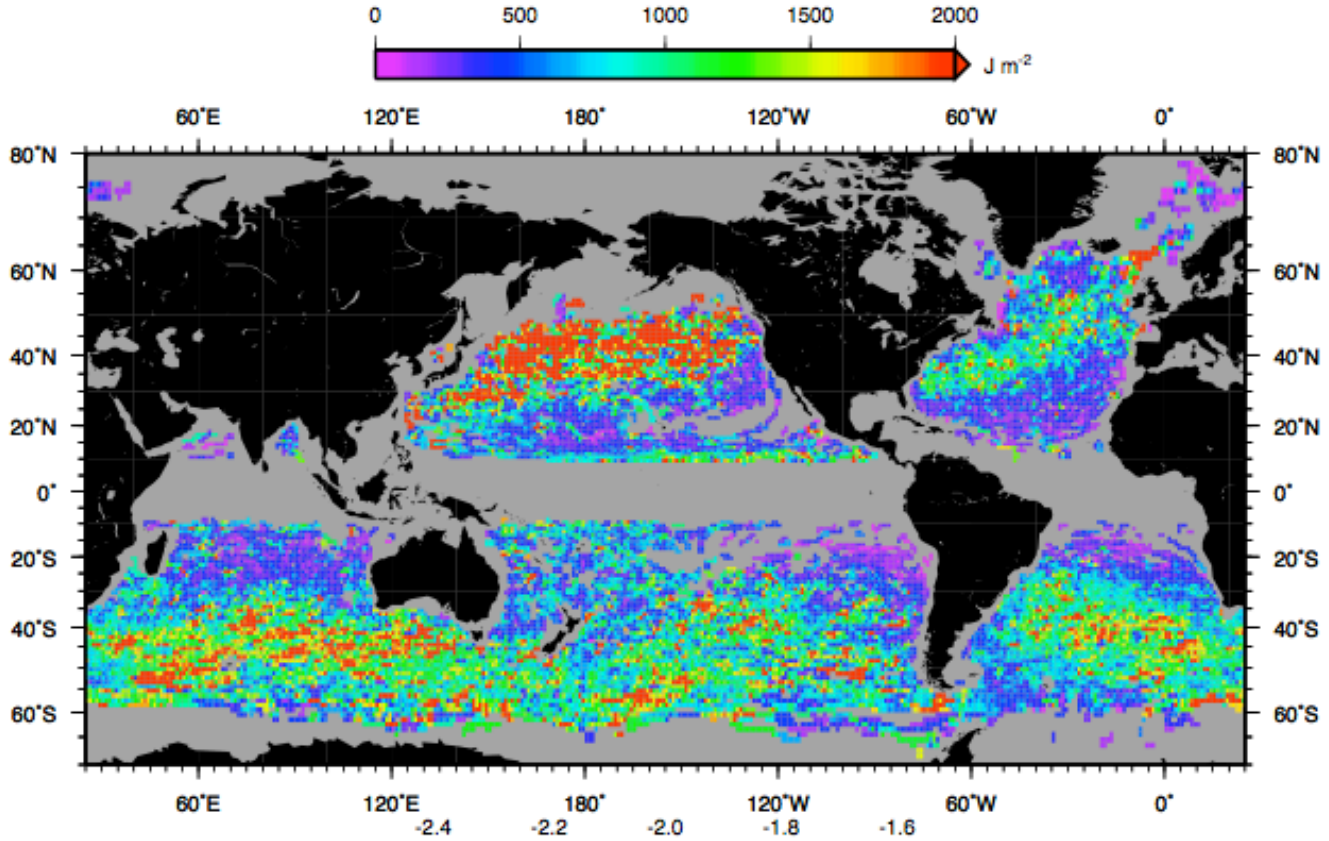
Coriolis in action in the ocean: Observations of Inertial Currents



DPO Fig. 7.4
D'Asaro et al. (1995)

- Surface drifters in the Gulf of Alaska during and after a storm.
- Note the corkscrews - drifters start off with clockwise motion, which gets weaker as the motion is damped (friction)

Inertial currents: relation to internal waves



Energy in inertial oscillations (which are wind-forced) as measured by surface drifters (Elipot et al., JGR 2010)